Multiculturalism, Migration, Mathematics Education and Language

Project Number: 526333-LLP-1-2012-1-IT-COMENIUS-CMP

ORNAMENTS IN TEACHING SYMMETRY

by Hana Moraová[*] and Jarmila Novotná[*]

INTRODUCTION

The following unit looks at the potential of multicultural content of ornaments of different culture and their potential use in mathematics classroom. What mathematical structures can be practiced using the cultural content of ornaments? What cross-curricular links does the unit bear? How can it help integration of migrant pupils into the classroom?

Piloting with trainees

The unit was first piloted on a workshop with Czech pre- and in-service teachers. In the workshop the participants were introduced to the issue of teaching mathematics in multicultural classrooms. The aim of this piloting was 1) to show pre- and in-service teachers how easy it is to add multicultural content in to lessons of mathematics, 2) to get more ideas for what mathematics is hidden in ornaments.

Consequently, the trainers presented a number of ornaments from different cultures and asked the trainees to pose as many mathematical problems working with these ornaments as possible.

Anticipated mathematical topics for development

Symmetry, rotation, translation, plane geometry, tessellation.

Other topics developed by the trainees: proportionality, linear functions, ratio, combinatory, least common multiple.

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[*] Faculty of Education - Charles University in Prague, Czech Republic.

Aims of the workshop

***For trainees*:**

- Investigating solving/learning strategies
- Posing problems
- Discussing these problems in groups

For trainers:

- Enrichment of mathematical content that can be used with ornaments
- Enrichment of repertoire of possible multicultural problems for mathematics lessons

Main piloting

by Hana Moraová and Jarmila Novotná

1. Description of the activity

The activity was based on the concept of substantial learning environments – SLE developed by Erich Wittmann (1995), namely the concept that "A good teaching material for teachers and pupils should be the one which has a simple starting point, and a lot of possible investigations or extensions." The simple starting point in this case was a number of ornaments whose origin was in different cultures (with the intention of allowing minority pupils to be heard, to present ornaments typical for their culture or home, to break the wall between home and school culture between mathematics naturally used at home and mathematics used at school – Meany, Lange, 2013). The trainees were invited to pose as many problems with the content as possible. Problem posing is an important component of the mathematics curriculum, and is considered to be an essential part of mathematical doing (NCTM, 2000; Tichá, Hošpesová,2010). It is an activity a mathematics teacher does almost on everyday basis when they need to supplement problems from the textbook.

Stage 1 The trainees

- Introduction to multicultural issues and intercultural psychology and their implications for mathematics classrooms
- Discussion of traditional way and typical tasks in teaching symmetry
- Activity – symmetries in letters in different alphabets, small and capital letters, symmetries in words

Stage 2 The trainees

- Activity –ornaments of different cultures
- Types of ornaments – symmetrical vs. asymmetrical, nature, geometry, line, tessellation, rosette

- Task: Pose a problem and/or develop a lesson plan and activities using your (or other selected ornament). What mathematical content is there?
- Present your problem/lesson plan to other trainees
- Discussion of the plans, selection of best activities

Stage 3a The trainees

- Prepare the final draft of the lesson plan to be piloted, prepare the needed teaching materials and aids

or

Stage 3b The trainers

- Choose one of the proposed activities
- Draw the final draft of a larger didactical unit (several lessons) which is flexible and can be adapted for use at different levels and in different grades
- Adapt the teaching unit to meet the needs of the selected classroom, prepare the needed teaching materials and aids

(This is the ideal scenario in case of pre-service and in-service training. In case of this piloting, the final draft was made by the research team/trainers as there was too long time between the in-service training session and the piloting at school, see the following text.)

Stage 4 Piloting at school

- The final draft is taught at a selected lower secondary school
- Immediate feedback from the learners (about 5 minutes)
- Post-lesson interview with the teacher
- Teacher's written reflection on the activity

Stage 5 Piloting in a selected school abroad

2. Assignments

a) Problems posed by trainees

- The Pythagoras theorem: measure and calculate with the triangles in the presented ornaments.
- Compare line symmetry, rotation and translation. What is typical for which ornaments?
- Find all the different geometrical shapes you can find in one ornament; name them and describe them.
- Study the concept of tessellation; find which ornament can make tessellation.
- Copy the ornaments on a square grid, look at their area. Use square grids of different scale, study proportionality.
- Calculate proportion of area of one colour.
- How much fabric with this ornament would you need to make one e.g. kilt (about tartan)?

- Find the generating element.
- How many lines of symmetry are there in a specific ornament?
- The least common multiple (in case of Indian line ornaments).
- How many beads are needed to make one segment of Native American ornament?
- How much band is needed to decorate a wall of certain dimensions?
- Patchwork and ornaments, what geometrical shapes are possible for production of patchwork?
- How many threads of each colour do we need to make one segment of tartan?
- Draw symmetrical ornaments, copy them from the original or create pupils' own ornaments.

b) Lesson plan

The trainees studied the ideas and agreed to build the following didactical unit. The unit was developed and elaborated in detail but for the needs of piloting was then adapted by the teacher to suit the needs of the School Education Programme, mathematics curriculum of the particular group and the needs of the children.

Note: For materials used during the unit see e.g. www.googleimages.com.

<u>*Lesson 1*</u>

- Title of the lesson: ORNAMENTS
- Revision of symmetry: look for lines of symmetries in different types of letters (appendix 1) – 10 minutes
- Lead-in: presentation on types of ornaments in different cultures (10 minutes)
- Main activity
    - show ornaments from different cultures
    - on one or two show the different types of symmetry and transformations
    - give each student one ornament and ask them to find all lines of symmetry
    - ask students to name and copy all symmetrical geometrical figures in the ornament
    - ask students to formulate conclusion about typical ornaments of a particular culture
- Homework: bring an ornament decorated object from your home, bring pictures of various ornaments from your holidays.

<u>*Lesson 2*</u>

- Lead in: present your ornaments, what types of ornaments are they, what line symmetries did you find?
- Main activity:

- Give each student one of the three ornaments (Celtic, Native American, Arabic rosette) and a square grid with different scales
    - Ask students to find all lines of symmetry in their ornament
    - Ask students to copy the ornament into the square grid
    - Ask students to count the number of at least partially coloured squares
    - Ask students to calculate the area of the ornament (taking partially coloured squares as covered squares)
- Follow-up: Copy the following chart in the whiteboard

scale	0.5 cm	0.75 cm	1 cm	1.25 cm	1.5 cm	2 cm
area						

What proportionality is there between the scale and the area?

Piloting at school

The Czech activity was piloted at ZŠ Fr. Plamínkové s RVJ in Prague in the 5th grade.

The research team closely inspected the Framework and School Education Programmes for Primary School Education in the Czech Republic (MŠMT 2013, http://www.plaminkova.cz/skolni-vzdelavaci-program) to see which of the topics listed in the above described proposal are suitable for this age group. Czech 5th graders do not yet have knowledge of symmetries and do not work with them explicitly, but are likely to have intuitive knowledge of them. In the 5th grade they learn to work in square grid and can build their preconcepts of plane geometry (area and perimeter). They have not been introduced to the concept of proportionality.

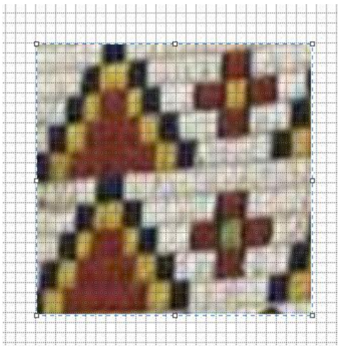
The decision was made by the team to adapt the teaching unit and to pilot two teaching units:

Lesson 1

Introduction to ornaments, discussion of ornaments, types, shape, differences in cultures, basic elements; discussion of Native Indian ornaments (made of beads).

Children were given square grid (0.5 cm) and a Native Indian ornament and were asked to copy it square by square (accurately), then they were asked to calculate the number of blue squares, size of blue cross, blue and yellow figure; however this could not be used as introduction to area because of the scale.

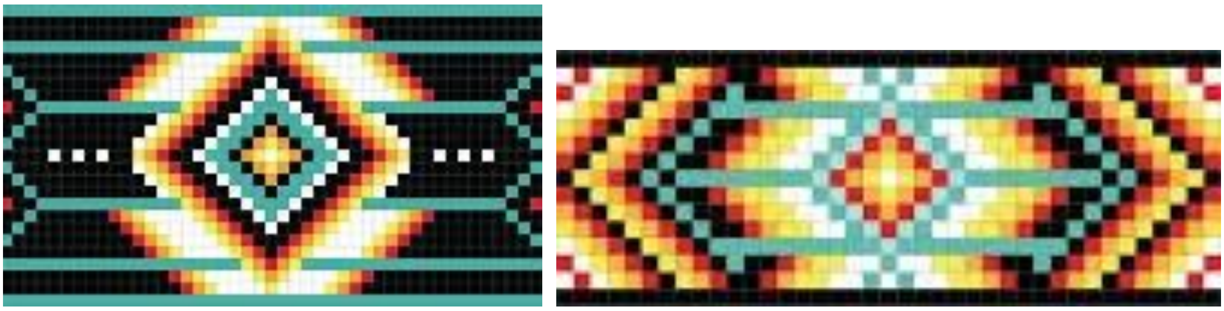
Children were given 1cm square grid and another example of a Native American ornament that had been embroidered not made of beads, i.e. it was made of rectangular, not square elements. Two part of the ornament had been copied on a square grid (one rectangle made of three times three squares). The children were asked to copy these figures into 1x1 cm square grid.



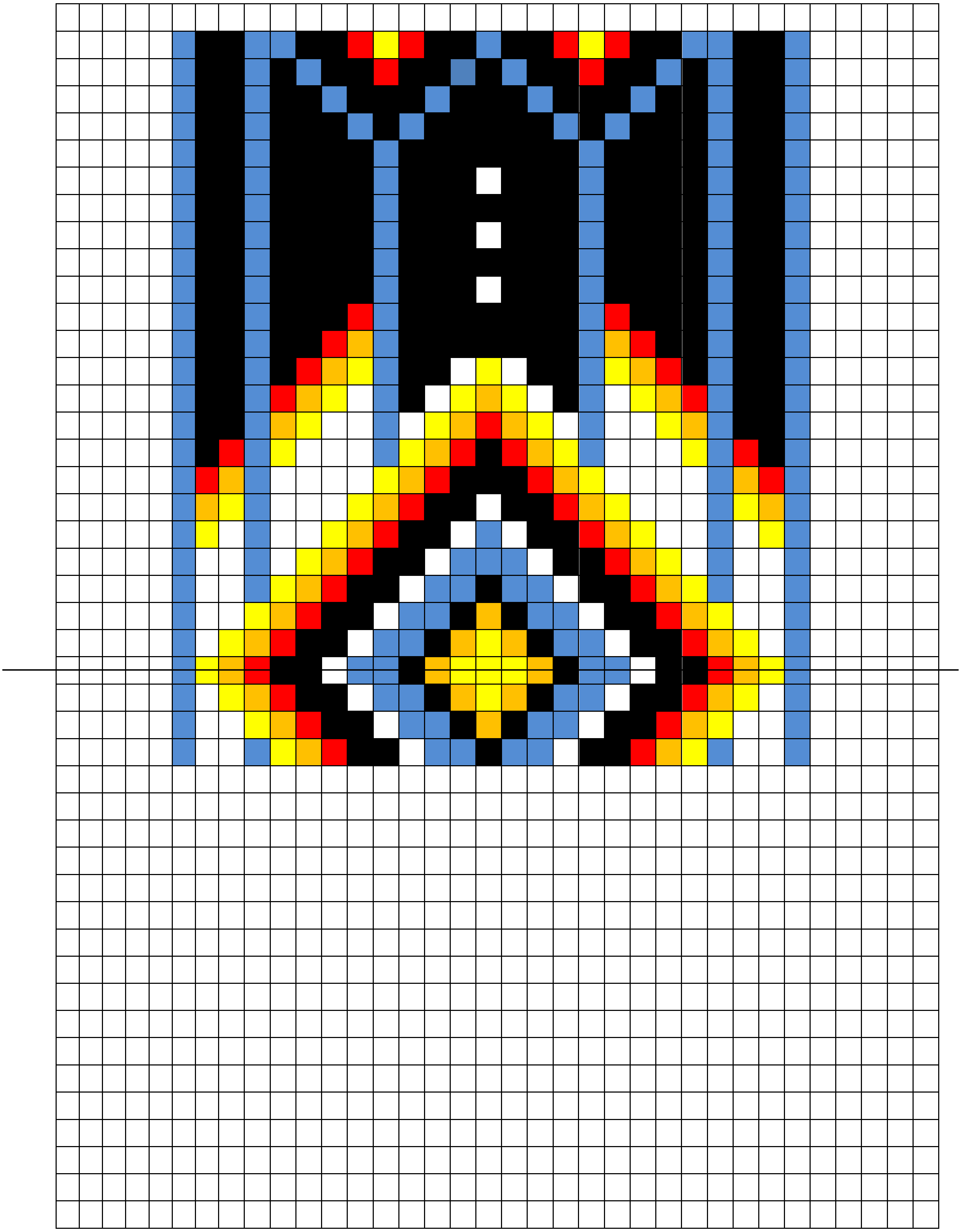
As the area of each square was $1cm^2$, the pupils could then easily state the area of different geometrical figures they had drawn (rectangle, 2 rectangles, cross, pyramid etc.). The same was done with perimeter.

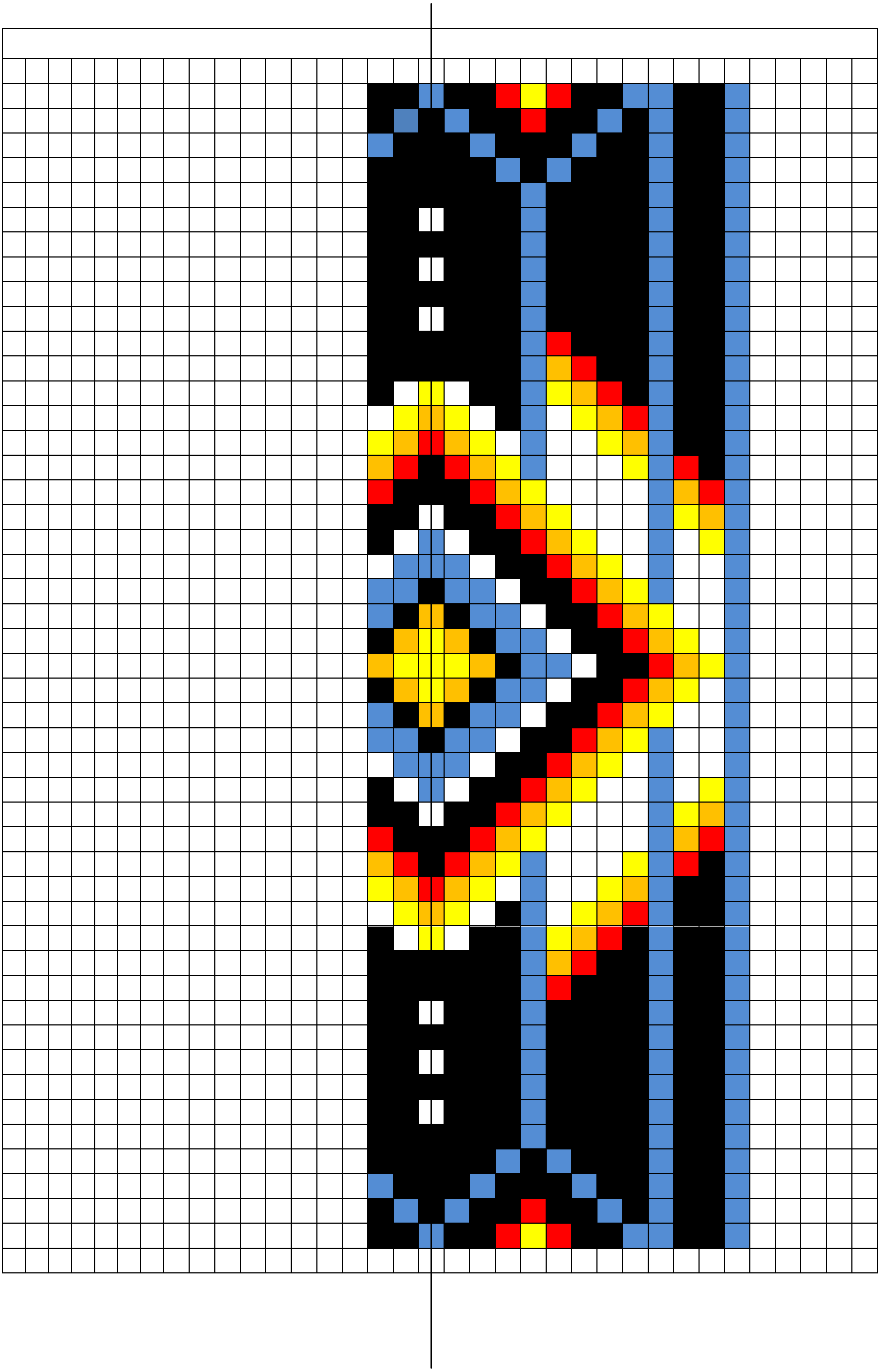
__Lesson 2__ – work on symmetry (a combined Math and Art lesson)

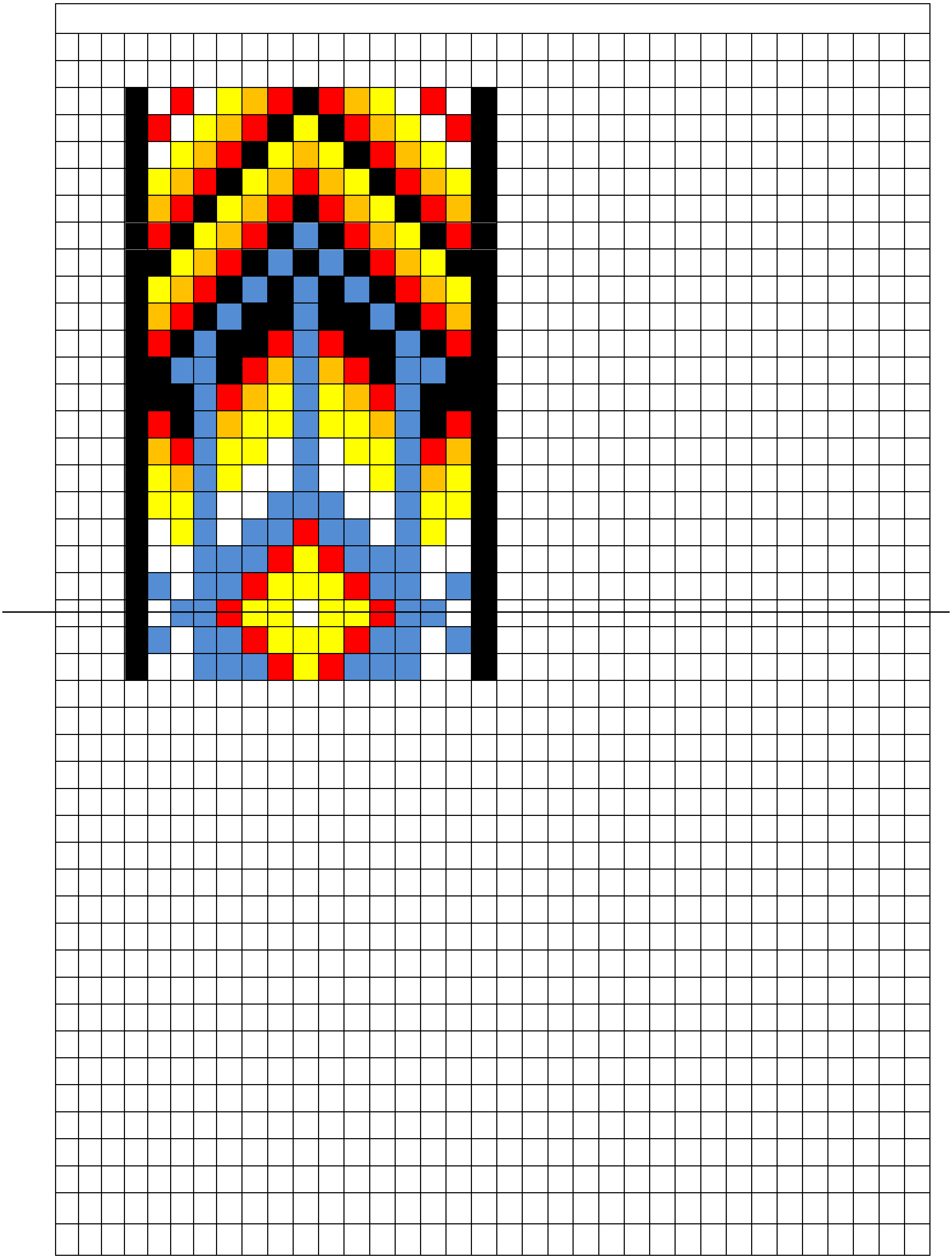
Children were shown two original Native American ornaments. They spoke about the figures they can see there.

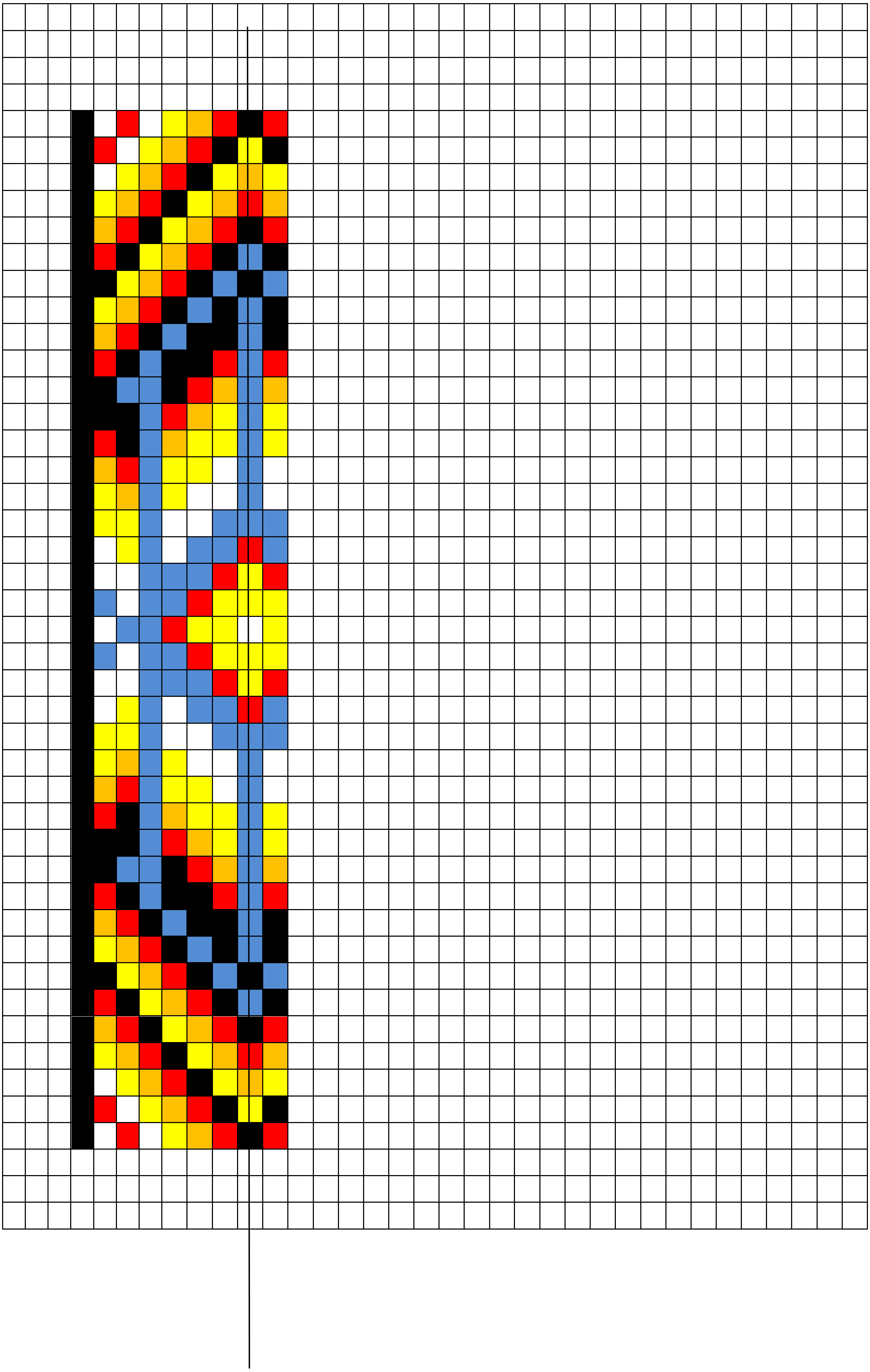


They were given a model of an ornament with line of symmetry indicated and asked to finish the ornament. An example follows. More models are available on the project website or on the DVD with the full text.









They used crayons, beads, threads etc. to create models of ornaments.

Task for homework: Look at ornaments you have at home. Where are they? What types? What shapes, colours, materials etc.? Copy them or take a picture of them. We will be working with them in the next lesson.

Comments by the piloting teacher

The teacher who was piloting this activity in general evaluated the teaching unit as motivating to her pupils. The pupils were occupied and working hard most of the two lessons. The materials allowed differentiation in the lesson (selection of less and more difficult figures, number of figures to state the area and perimeter).

The teacher suggested that with this age group only 1cm square grid be used in all activities as it makes counting different squares more mathematically more meaningful from the very beginning. She warned that in the first task pupils need to be alerted to the fact if they do not begin with the central blue figure but start from counting the number of red squares and drawing the outlines of the red rectangle, they often find out very late that their original outline had been incorrect and does not leave space for the central blue cross. If this happened pupils were not much motivated to start from the very beginning.

Second piloting

by Antonella Castellini, Lucia Alfia Fazzino and Franco Favilli[**]

The a priori analysis

The context

The activity was both designed for and developed by a group of students from two different classes of the "*Istituto Comprensivo 1*" in Poggibonsi (Province of Siena) during the weeks of flexible teaching. In short, during these weeks the classes are open to carry out various disciplinary or interdisciplinary activities and to develop projects outside the school. The group consisted of 15 to 18 pupils of two second year classes of Lower Secondary School.

The aims

The topics dealt with in the teaching unit allowed to retrieve previous pieces of knowledge, but also to see them in a different and certainly more creative way. Significant added value was represented by affectivity, since the activity urged the reference to the typical cultural values of the students' native country. The teaching unit topics allowed the introduction of new teaching methods, such as problem solving, which so far had not been used for the development of mathematical skills.

The reasons for this choice were basically four:

- to look at reality with mathematical eyes;
- to develop intercultural education, accompanied by the desire to let students know about other cultural roots;
- to develop a positive attitude to mathematics, through meaningful experiences (as suggested by the Italian National Guidelines 2012);
- to describe, name and classify geometric figures, identifying their relevant elements and symmetries, also in order to make all students able to reproduce the figures (as prescribed by the Italian National Guidelines 2012).

The design

The methodology adopted was that of the workshop, intended not just as a physical place, but as a classroom activity where doing and thinking are closely related. The workshop is understood as a teaching situation where the meaning of mathematical objects are constructed through experiences that are rich and stimulating for the students themselves.

All the activities of the teaching unit were designed with reference to the isometric transformations, a topic partially introduced in the previous school year. The topic was addressed by the use of a mirror: by putting a drawing or an object in front of

[**] CAFRE – University of Pisa, Italy.

the mirror and by the observation of the reflected image, the students were able to discover the key features of the axial symmetry. (Photo 1)

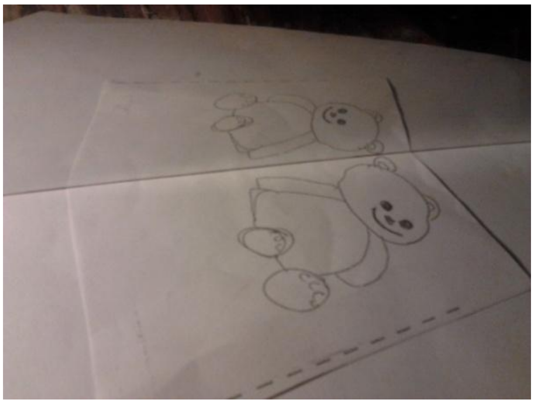


Photo 1

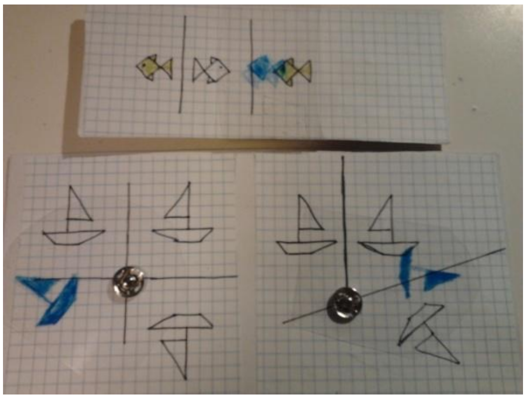


Photo 2

The next step was to build (as the composition of symmetries with parallel or perpendicular axes) the other isometric transformations – translations and rotations – and identify their basic features. Afterwards, all the students had to build some dynamic models that allowed them to represent these geometric transformations. (Photo 2)

The object-action connection allowed the students to be free to design and to interpret. Indeed, it is for this reason that it becomes important to see, observe, interact with a dynamic, non static object.

The static nature limits the student, forcing them to look at one aspect and does not help them to analyze the situation from different points of view. Besides, it does not stimulate students' curiosity, and above all, it does not allow them to speculate, or even less, to argue, thus excluding a substantial part of those processes that are fundamental in the formation of mathematical thinking. Therefore, besides the use of the dynamic models already known by students, a new activity with the use of the room of mirrors was planned.

The teaching unit

Lesson 1

In the first lesson two ornaments - a frieze and a rosette - are delivered to the students, who are divided into 4 groups of 4-5 students each. Students are then asked to analyze them by using the flat mirror in order to identify what kind of isometric transformations allowed the creation of the ornaments.

After a brief plenary examination, the students have to choose a spokesperson who will present to the other groups the geometric transformations identified in their own group. The purpose of this activity is to reflect on previous knowledge and to compare the results of the different groups, with the main objective to make them infer conjectures and to learn how to argue with other about them.

Frieze group

Students begin to observe the frieze using the flat mirror: "At first we put the mirror horizontally towards the décor (Photo 3), but while observing, we realized that such a symmetry was not possible because a frieze is usually very long. So, we put the mirror vertically to the drawing (Photo 4) and we observed that there is an axial symmetry when considering the square as its module. If we consider the squares a, b and c, it can be clearly seen that there is a double axial symmetry with parallel axes. Then, looking more closely, we realized that if we consider figure a and b as a whole then there is also a translation to the right. (Photo 5-6-7)"

Photo 3

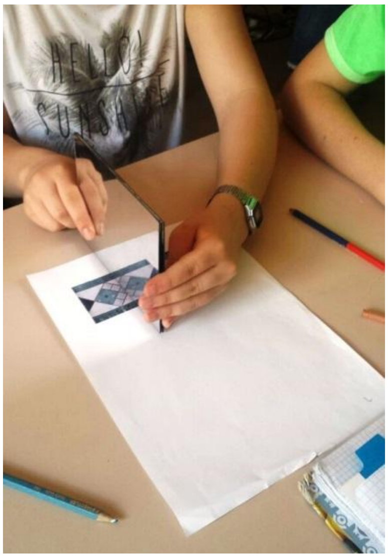


Photo 4

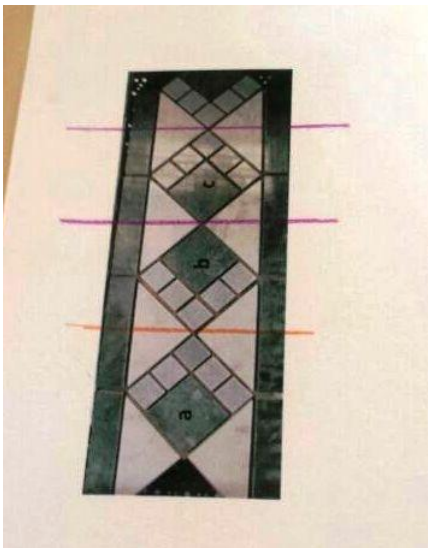


Photo 5

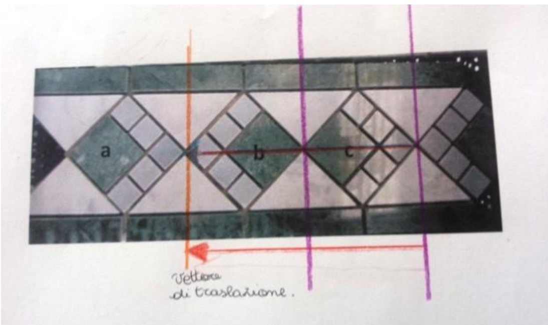


Photo 6

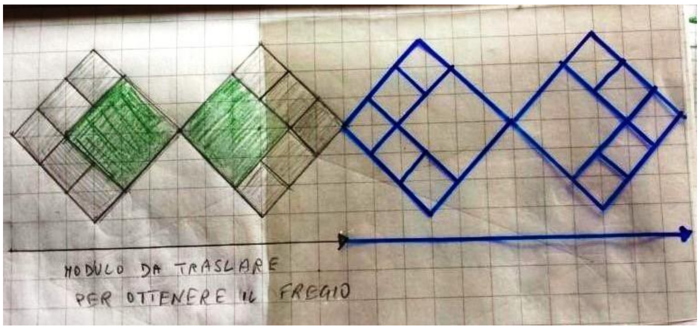


Photo 7

At this point they asked themselves the question "but how long is the translation vector?"

With the help of the dynamic model they have verified that the length of the translation vector is exactly twice the distance between the two parallel axes that gave rise to the movement.

Décor group

Pupils' comments are directly quoted: "In this pattern we saw immediately that there are axial symmetry with incident axes (photo 8). Then A. pointed out to us so that there is also a rotation, because it is produced by the composition of two symmetries with incident axes. So we decided to draw the axes of symmetry and we found the centre of rotation. To check the rotation we took the acetate and had our dynamic model. (photo 9)

Photo 8

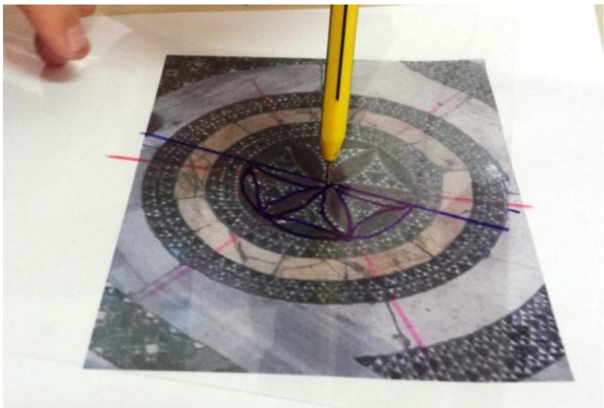


Photo 9

Interesting is the fact that the pupils show on the decor the module and the axes of symmetry to explain their reasoning and resort again to the dynamic model for dissolving any possible doubt.

In this first lesson, to get confirmation of what had been observed, it has been very helpful the use of the mirror and the dynamic models that the pupils had built.

Once reviewing the various isometric transformations is finished, students are asked to do at home, individually, a written summary of the work done in the class (log book), and to search for and bring the day of the next lesson, objects and / or fabric available in the house that contain decorations and are proper to their country of origin or taken from visited countries.

Lesson 2

Both the objects and the fabrics are made available to students. Each group selects the item it prefers. A fabric from Senegal (the most successful in students' eyes, perhaps because very colourful and with different types of decorations) and another fabric that a female student uses at home to cover the sofa, this also pretty colourful and with very regular decor. Students discarded different laces (many of which crocheted by grandmothers...) as well as ceramic plates and boxes, actually less numerous and not very colourful.

This task is assigned to each group:

1. motivate the choice of the object;
2. identify the isometric transformations with the help of the mirror plane;
3. reproduce the chosen ornament on two squared sheets that you have been given;
4. identify the generating pattern of the ornament;
5. present to other groups the chosen decor, providing each of them with the generating pattern and the instructions to create it.

Clovers group

The members of the group that has chosen the ornament with clovers (Photo 10) motivate their choice with the fact that it was "easy and pretty." In fact, as they write in their report, they were initially mistaken: "We thought only of simple symmetries in both in the petal and in the ornament, from square to square, but then, looking better with the mirror, we realized that there was the small flower stem and that it was not an axial symmetry, then, but a central symmetry. (....) For this we used the acetate with the popper, to get it better understood."

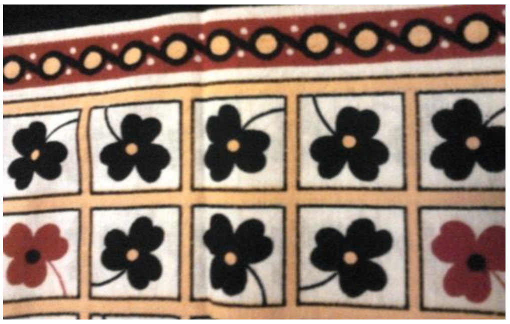


Photo 10

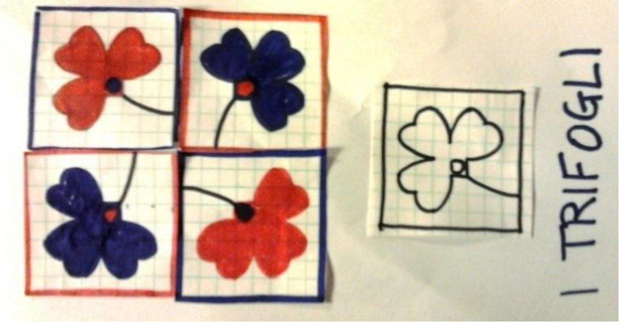


Photo 11

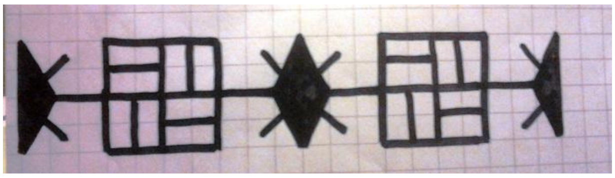
Photo 12

The work done (Photo 11-12) provides clear evidence of their research and shows how the mirror helped them to better identify the isometric transformations in the decor, but also how much the use of dynamic patterns was useful to view the changes.

From the base pattern, with successive rotations of 90 degrees, they were able to represent the decor of the fabric. It is interesting to point out the choice of the colour that the group made to highlight the two figures matching in the central symmetry.

<u>Small frame group</u>

Photo 13



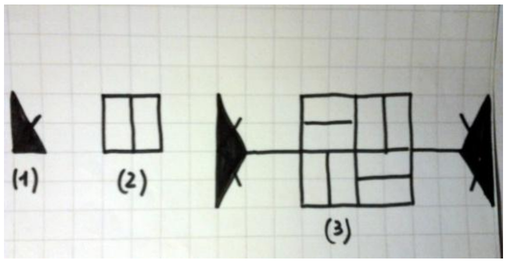


Photo 14 **Photo 15**

The group that chose the decoration with small frames (Photo 13) states it did it because "it looked like one of the small frames that were made during the elementary school years". Pupils have reproduced easily the pattern highlighting immediately the translation (Photo 14) which allowed them to reconstruct the entire decor. But also on this occasion, after a moment of euphoria for the speed of their execution, students have paid attention to the central decor that is "what is made of small rectangles." The pupils have realized that there were other symmetries: in particular, a pair of central symmetries or a series of four 90° rotations. They were then sure that there was nothing else and started to write a sequence of instructions to make their mates able to reproduce the little frame. While writing, however, they realized that also the other decor - "the black star type one" – appeared actually to have the same property of double central symmetry. There have been, therefore, many discussions: "How do we do it? Have we to give three different instructions? One for each of the two patterns, and one for the small frame at the same time? And what then is the generating pattern?". Pupils agree to proceed this way and give three different instructions by providing classmates with the three patterns shown in the photo: two patterns to recreate the central motifs with the rotations and the other to make the translation. (Photo 15)

<u>Lozenge group</u>

Again the choice is motivated by the pupils saying that "they are very colourful, but there are curves and straight lines, that is two different ways of decorating". But it is this situation that makes it difficult for the group to represent on the paper the fabric design (Photo 16) and necessary a little help from the teacher. The pupils then realize once again that only four rotations of 90° are needed to recreate the ornament and easily identify and compose the basic pattern. (Photo 17-18)

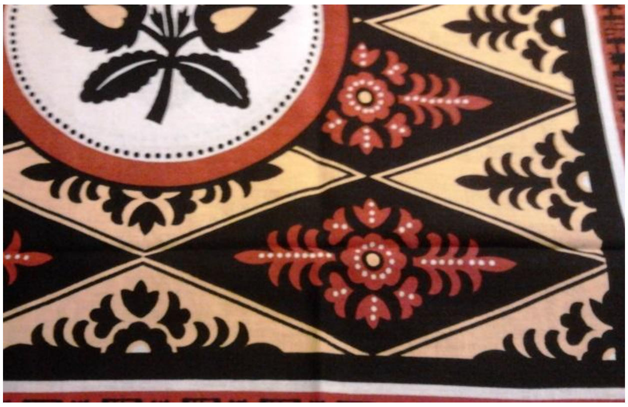


Photo 16

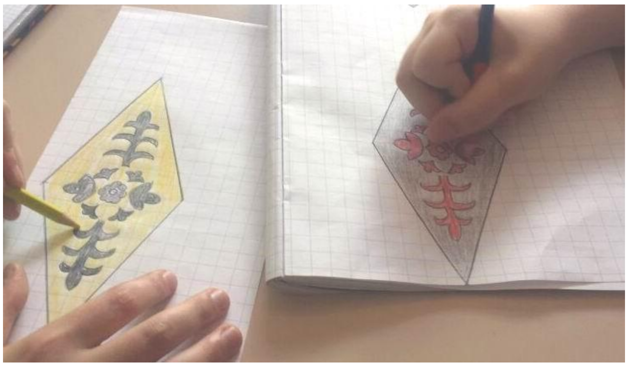


Photo 17

Photo 18

<u>Rosette group</u>

The group motivated the choice of the rosette (Photo 19) by saying that it reminded them of some Christmas drawings. The ornament undermined the students: they failed to identify the starting pattern. They could see symmetries, but only in two pairs of the ornament elements and they recognized the rotations, but could not explain how to make use of them. They discussed a lot about "separating the two designs, the one with a small square from the other. They tried to make use of a pattern, then they tried with another and finally decided that "we can work with three patterns to reproduce the entire rosette: two of them must rotate four times by 90°, the third one must rotate 8 times by 45°". (Photo 20-21)

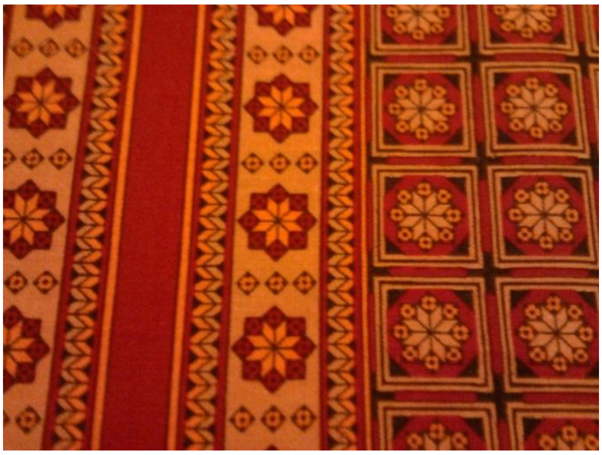


Photo 19

Photo 20

Photo 21

Lesson 3

A new activity was introduced, which could be titled:

From the room of mirrors to proportionality

Consider the patterns you have made in the previous lesson and which originated the rosette ornament. (Photo 22)

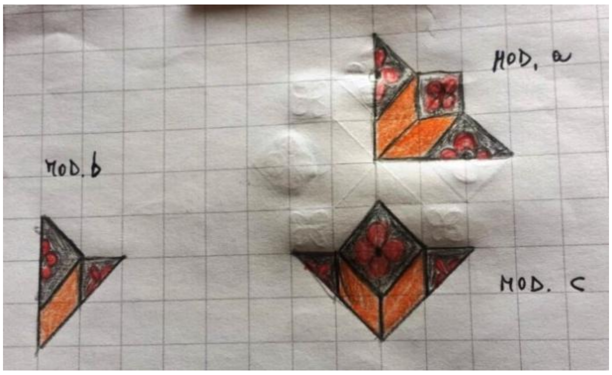


Photo 22

Place the patterns, one at a time, in-the mirror room and tell how many of them are needed to complete the ornament. (Photo 23 a-b-c)

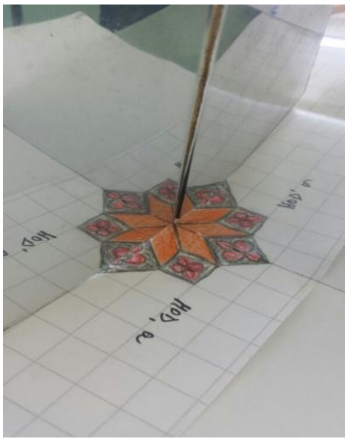


Photo 23 a

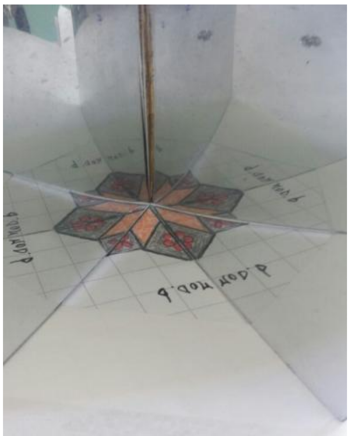


Photo 23 b

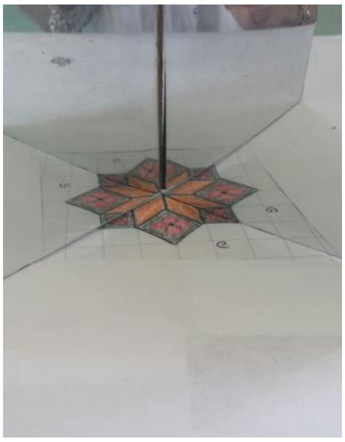


Photo 23 c

Pupils begin to put the mirrors over the pattern at an angle of 90°. In such case they see four images (three are reflected and one is the real one) forming a flooring, that is, an entire decor. Having then put the other pattern, they realise that “with the 45° angle, however, we get 8 images, so they doubled. And yet the angle is decreased, more precisely it's halved!”.

This remark stimulates the curiosity of the students who begin to try other pieces of the ornament. "Let's try with half rosette, we certainly get two images only". Then they come to say that if the angle of the mirror room decreases, the number of images increases.

The teachers therefore to get deeper into the topic and suggest pupils to use the protractor to find the angle between the mirrors and to place in the room of mirrors a thin object such as, for example, a pencil. They also suggest to build a table with the angle between the mirrors and the corresponding number of images obtained. The angle will have a measure equals to a sub-multiple of the full angle. In the table, then, there are going to be not only the pairs (90, 4), (180, 2), (45, 8), but also, for example, (30,12), (40,9). The table will clearly show the relationship *angle width* ←→ *images number* because it is easy to see that if the angle halves, becomes a third ..., the number of images doubles, triples .... Pupils can, therefore, observe that the product of the angle width by the number of images is constant and equal to 360°, the full angle.

In this way the students have then discovered intuitively, but at the same time rigorously, the law of inverse proportionality!

Afterwards, the teachers decide to ask pupils to represent the data in the table by points on a Cartesian plane and connect them. Pupils can, then, realise, that the points can be seen as elements of a curve that they do not know yet, a branch of hyperbole.

It is here that the student B. asks: “Why does this graph start at 10°? If I close the room, that is if the angle is zero, what happens? I do not see anything so I don’t get any figure, the images are zero ... but then it does not work ... there's something wrong”. Here it is how the one student’s doubt becomes a resource for everyone! The teachers suggest then pupils to enter a piece of string in the room of mirrors and look attentively what happens when closing mirrors slowly. The closure action enables students to understand that the images are not zero but infinite “in fact, we do not see them because they are inside!”. Once again the dynamism of an object leads to examine an important limit case it would not be easy to deal with and understand just by arithmetic, since the division by zero is impossible. In this way, on the contrary, pupils, through this operation that have verified to be impossible, can grasp the idea of infinity.

Lesson 4

Art tessellations

The students have already worked on the tessellation of the plan and know what are the regular polygons that make it possible and the reason why. A slightly modified version of this activity is then proposed to the students, with the aim to unleash their "artistic creativity".

Students are asked to cut out a part of a square and place it on the opposite side. In this way they get a pattern that, by subsequent translations, creates a flooring. The same activity can be proposed using other regular polygons, such as the equilateral triangle or the parallelogram. The creativity of each student will transform the pattern he/she has into a subject that will be the "hero" of this new and very personal flooring. (Photo 24-25-26)

Photo 25

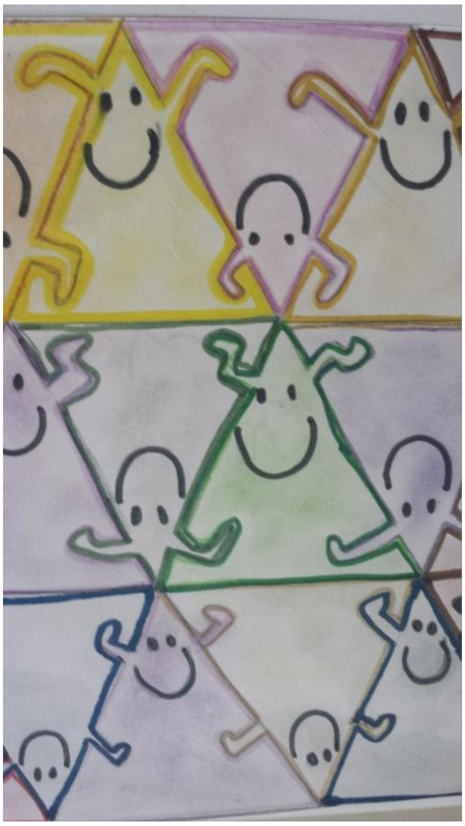


Photo 24

Photo 26

The activity was very much enjoyed by the students who, after a temporary confusion due to the actual construction of the pattern, had fun creating beautiful floors, while showing imagination and artistic sense.

Unlike what happened with the square and the parallelogram, the use of the triangle as the polygon to start from proved to be difficult. "Where was the cut part to be placed in order to obtain a flooring? Is it ok to put it on any of the other two sides? Or, is it necessary to place it on the same side where it has been cut from?" These questions arose spontaneously and led to a good discussion that was developed with good arguments. Once again a context of emotional and meaningful reflection encouraged spontaneous and interesting questions that, properly managed by the teacher, can give the opportunity to break new grounds or retrace paths already experienced, but from a different perspective, thus developing a continuous reconstruction of knowledge.

Conclusions

We are firmly convinced that there is a strong need to change attitudes towards mathematics and that as the Italian National Guidelines 2012 say, in the classroom "a positive attitude towards mathematics through meaningful experiences" should be promoted. Therefore, more than accumulating knowledge by just passing a number of notions and information often not interlinked or interrelated, teachers should try to stimulate students' aptitude to pose problems that can increase their motivation and foster discoveries.

The teaching unit described above falls within this framework, making use of workshop activities in such a way that learning is really centred on students, on their needs and on their characteristics. The student is the investigator and as such, acquires the ability to identify, accept, confront and solve new problems, both individually and in groups.

The unit development is based on three methodological cornerstones:

1. to set context problems;
2. to foster questions;
3. to work in groups so that the heterogeneity of the students is a resource for the entire class with a view to get more and more inclusive learning.

Many are the problems teachers have to face such as dealing with unmotivated students, working in socially non-homogeneous and multicultural classes and with students of different cultures… It is therefore necessary to design educational courses that allow students to see the reality from different perspectives and also to develop greater self-awareness.

The teaching unit allowed teachers to meet today's students needs without sacrificing the teaching of the basic concepts of the discipline. Even though mathematics is often seen as an abstract subject, it could instead become somewhat closer to the students and to their reality. The use of everyday objects, also linked to different cultures as the decorated fabrics from Africa, gives the subject an affective meaning which should not be underestimated. Even the realization of ornaments, an activity that makes the student free to experiment and indulge within its fantasy, offers an emotional dimension that is important because learning is difficult if the sphere of emotions is not positively affected. On the other hand, teamwork allows students to learn how to defend their conjectures and at the same time, to accept changes when their classmates' arguments are clear and justified.

The whole activity, therefore, is based on fundamental aspects of the learning process; in fact, it requires students to be active, constructive, collaborative, contextual and process reflective. This way, it provides their students with excellent cognitive skills.

Third piloting

by Andreas Ulovec[***] and Therese Tomiska

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting of lesson 1 during a regular mathematics class (50 minutes) in the 6th grade, and lesson 2 during a 50 minute class using field work as a teaching method. Eight students (age 17-18), three of which are migrant students, attended the class, of which lesson 1 was observed, and lesson 2 was video recorded and observed by a member of the Austrian project team.

Classroom piloting

The teacher introduced the topic by bringing several objects with Japanese, South African, as well as US motives from her private possession into the classroom for lesson 1. The students formed groups of two and were asked to look for symmetries as well as for different geometrical figures, and finally compare the different kinds of figures and symmetries that they found in the objects from the different cultures. Each group then shortly presented their findings in front of the whole class, and the other groups wrote them down into their notebooks. At the end of the lesson, the teacher asked the students to bring ornaments or pictures of different cultures' ornaments into the next lesson, as suggested in the proposal. The students argued however that only very few of them (or their families) actually had suitable

[***] Faculty of Mathematics - University of Vienna, Austria.

ornaments and pictures at home. Repeating the lesson with more objects from the teachers' collection was seen as not very interesting by both the teacher and the students. The students then came up with the idea to go out into nature and bring pictures of symmetries or geometric figures that are found on flowers or plants instead. The teacher argued that if symmetry in nature would be interesting for the students, it would be better to actually make a field work session out of lesson 2, instead of just looking at the pictures. It therefore was decided that lesson 2 will be modified, and students will go out together with the teacher, look for symmetries in nature, and take photos for later discussion of symmetry and scale in class (this last part, i.e. back in classroom, was not part of the piloting).

Photos 1-3. Patterns from Japan, South Africa, and USA

Lesson 2 started with the teacher reminding the students on the different kinds of symmetries and figures, as well as special angles (e.g. from Fibonacci numbers). Then the teacher and the students went out into a field near the school to look for the occurrence of symmetries and geometric figures in both natural and artificial objects. Students first looked for the occurrence of certain angles on plants. Very soon they realized that 137.5° was a very frequent angle on a number of species of plants, a fact that impressed the students very much. Students took pictures of the objects to use in the next lesson.

Photo 4. Looking for certain angles on a thistle

The unit continued with the students looking for symmetries, particularly for mirror-symmetry. There, the students were mostly able to state that the object does actually show some kind of symmetry, but were not always able to name the kind of symmetry concerned. So fairly often, the students pointed out symmetries and the teacher explained the particular symmetry on this object.

Photo 5. Mirror-symmetric blade of grass

Students then started a discussion about how exact these symmetries actually are. The teacher used this opportunity to point out that real objects (regardless of whether they are artificial objects like the ones she brought into the classroom in lesson 1, or whether they are natural objects like grass) are never exactly symmetric in a mathematical way, and that this is where modelling comes into play.

At the end of the lesson, also artificial objects (advertising pillar, patterns on t-shirts) were checked out, and students and the teacher discussed whether the patterns or the form of the pillar have cultural and/or practical reasons. Several of the shirt patterns were photographed; they came from different cultural backgrounds without the students (according to their own statements) having known this fact when they bought the shirts. The teacher gave as homework for all students to find out what cultural backgrounds the photographed patterns have and what their cultural relevance is.

Photos 6-7. Cultural patterns on students' t-shirts

The lesson ended with the class being back in the school building, where the homework assignment was repeated.

Conclusions

The piloting showed that even if the unit is modified and – at least superficially seen – moves away from the intercultural aspects, these aspects can be easily brought back into the minds of the students by referring to everyday life objects and their cultural connections.

Conclusions from the three piloting

by Hana Moraová and Jarmila Novotná

The proposed and piloted activities are of strongly multicultural nature. The teachers may use printed materials, materials downloaded from the internet or use everyday objects and motifs that surround us. Whatever the form, the use of these materials activates the pupils, motivates them to creative thinking and to looking for relations, it broadens their perspectives. The fact that very different ornaments are typical for different cultures and that these ornaments are often used to decorate everyday objects allows minority pupils be heard, bring contents from their own culture and motifs from their own homes into classrooms, it allows the teacher to show that mathematics is universal.

The experience from the three piloting shows that the materials can be used in a very flexible way. They can be adapted to the needs and knowledge of different groups and individual pupils. They can be used directly in the classroom, in the form of various projects, individual work outside of school. They are of strongly cross-curricular nature and can be used simultaneously in several subjects. The proposed environment and activity meet the criteria of Wittmann's substantial learning environment (1995).

The piloting show that if the activities are well planned and developed, they allow inclusion of pupils with different cultural backgrounds and traditions but also with very different interests. If they are given the chance by the teacher, each pupil will find their "cup of tea" and moreover can bring their own experience for the benefit of all. It is up to the teacher how the activity will be presented to pupils and how much freedom they will be given while working on it.

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Appendix 1

Czech **Matematika, MATEMATIKA**,

Hebrew **הקיטמיתמ**

Chinese **數學**

Japanese **数学**

Russian **Математика, МАТЕМАТИКА**

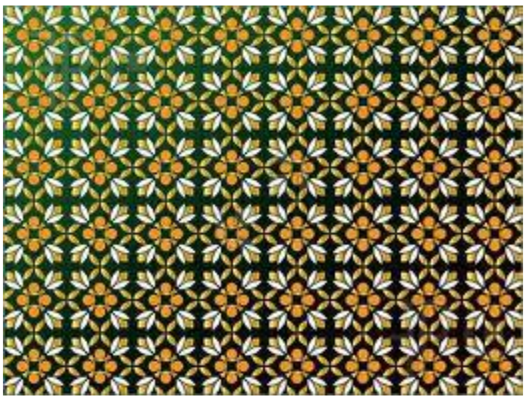
Greek **Μαθηματικά, ΜΑΘΗΜΑΤΙΚΆ**

Persian **تایضایر**

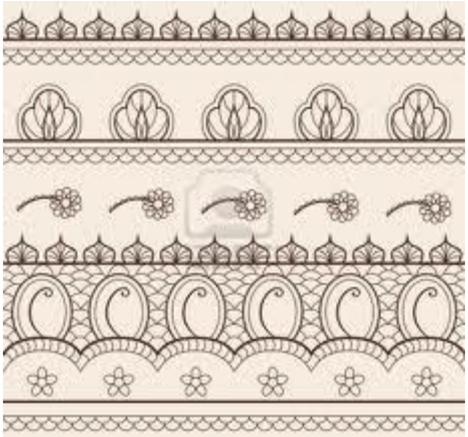
Appendix 2 – Ornaments from www.googleimages.com

Arabic ornaments





Indian ornaments



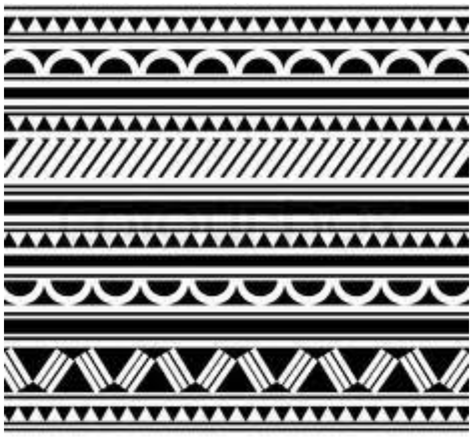
Gipsy ornaments

Moravian ornaments

Amerindian ornaments

Aborigen ornaments

Polynesian ornaments



Scottish ornaments