PUTTING BINS IN OUR SCHOOL'S YARD

by Charoula Stathopoulou* and Eleni Gana*

INTRODUCTION

This activity, based on a needs analysis, which was undertaken in an earlier phase of the project, intends to respond to the teachers' explicit request for teaching resources and provides an example of a good practice when teaching mathematics in multicultural/multilingual classes in secondary education.

The activity is informed by the broader research regarding language's role in mathematics teaching in multicultural/multilingual classrooms as well as by the recent literature for inclusive pedagogical initiatives regarding students with different cultural and linguistic backgrounds.

Specifically, the design of the activity adopts the methodological framework of a task-based approach (Willis 1996, Leaver and Willis 2004, Puren 2004). Through the activity **"Putting bins in our School's yard"** students were asked to answer to an existing, real life problem that emerged in their school environment. Finding a suitable space in the school yard to place recycling bins constitutes a real and recognizable issue for students' involvement in mathematics and provides them with the motivation to invest in the teaching and learning processes. Therefore, the mathematical situation is expected to make sense for all students; it does not also rely on well-defined mathematical procedures but instead allows for students' agency and

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collective negotiation of the mathematical notions (e.g. proportionality) and techniques (e.g. measuring). The very design of the activity sustains rich opportunities of communication between classmates and between students and the teacher in order for the class to reach a reliable solution. Students from different cultural and linguistic backgrounds could contribute with their own mathematical and discursive resources and make connections between informal and formal mathematics. In addition, in the context of students' joint action and synergy foreseen in varied phases of the activity, the different experiences and ways of dealing which the students bring to the group could be appreciated, and more equal relationships among peers could eventually emerge.

The technique of measuring and the notion of proportionality constituted the main mathematical content of the activity. The measurement – a procedure for the determination of an object's/ phenomenon's size – is an important mathematical process for determining the world around us, and helps to better control one's environment. The concept of proportionality is another basic mathematical concept, which runs through all matter of mathematics in compulsory education in various versions: scales, similarity, linear equation etc.

Workshops: Piloting with trainees

Before the final planning and the implementation of the activity (main piloting), there were three workshops, in which teachers of mathematics in secondary education (10 teachers) and one regional consultant for pedagogical issues in the lower secondary education took part.

The first workshop focused on language and cultural issues involved when teaching mathematics in multicultural/multilingual classrooms. The results of the needs analysis which preceded it showed that most teachers, although they taught mathematics in multicultural/multilingual classrooms, had not been trained accordingly. Therefore, in the first meeting there was a discussion of strategies which promote the language origin and the cultural experiences of students as teaching resources, as well as strategies that support the transition from everyday speech to mathematical speech. In the second meeting, the teachers were introduced to the main idea and the basic procedural phases of the activity. We discussed aspects of the mathematical content, its relation to the curriculum and the challenges that could present for the students of different linguistic backgrounds. During the third workshop we reached the final decisions of the activity's design. The teachers contributed to the decisions about: the phases of the project's implementation, time estimation in each phase, students' expected reaction on the activity and possible strategies suitable for the activity's implementation.

The design of the activity: Brief Description

- ***The Rationale***

A task-based teaching:

We selected to use a task-based activity, since through a task-based instruction students are expected to make sense of a mathematical situation for which no well-defined procedures exist. It is expected to also create rich opportunities for mathematical communication where probably language's issues could emerge.

The present activity is based on the existent ongoing discussion among students and teachers about the lack of efficient recycling policy in their school and how they could respond to this situation. So, it is about a real life problem that demands mathematical concepts and techniques, thus it is expected of all the students - since it is challenging for them - to be engaged looking for solutions and developing a suitable model.

♦ ***The task***

We present to the students a formal letter send by the Municipality concerning the official intent to provide rubbish bin/recycling bins to the school: students must respond through considering the space possibilities of their school yard, reasoning for the number of bins as well as for their choices concerning the bins' positions in the school yard.

♦ ***The students and the school***

The activity was designed for piloting - 1st piloting –in the 6th Gymnasium in Volos in the 7th grade (1st grade of Gymnasium) classroom, by the mathematics educator Ioannis Fovos. Out of a total of 22 students of the class, the 6 are Roma students and were bilingual using Romani, an oral language without written code, at home.

♦ ***The objectives***

- To connect mathematical notions and techniques in order to explore social issues and provide arguments for real life situations. Students will have the opportunities to use multiple solution strategies. For the implementation of the above, students were expected to use: space notions, measuring, similarity, proportion etc. Furthermore, it was expected that students would use propositions for locating, such as: down, up, above etc., probably in different ways, since, through previous research in the spot we have noticed that Roma people have a different space coding connected to their cultural peculiarities that is also depicted in the propositions they use.
- To communicate verbally and/or in other modes their ideas and their own ways of acting mathematically and to negotiate their reasoning socially, that is in the group, in the entire class, when addressing a formal agent, using different register and multimodal ways.
- To be actively involved in exploring different discursive patterns (genre, vocabulary and grammatical structures) when using and communicating mathematics in different contexts.

Main piloting

byCharoulaStathopoulou, Eleni Gana and Giannis Fovos

<u>The implementation of the activity</u>

The project was first piloted in a lower secondary school (6th Gymnasium) in Volos city.

1st phase: initial information *(10 minutes)*

The teacher informed the students that their class was selected for the implementation of a new type of mathematical activity and that the whole procedure was going to be videotaped. Almost all the students were challenged by this perspective, with the exception of one Roma student (a girl) who declared that she did not want to take part.

2nd phase.

a. Reading the letter *(20 minutes)*

The activity started with the teacher reading a letter sent (or supposed to be sent) by the administration and according to this, the students had to locate the suitable places for putting the bins in the school yard and to justify their choices. After finishing the reading, the teacher asked: *what do you think could be our first step for solving this problem*?

b. How to put the bins? *(20 minutes)*

Some students' characteristic answers on the above question:

- to take a picture from the roof of the building;
- to make a plan like this, showing the plan of the first floor that was next to the door of the classroom.

Students agreed with this suggestion and started to think how to do it. They realized that measuring the sides of the schoolyard was needed.

The teacher asked: since the measuring is going to give the real dimensions how could these be depicted on paper? Students answered that they knew how to do it although they could not remember the term 'scale'; a term that was reminded by the teacher. Then the teacher informed the students that they would move to the schoolyard to measure its sides and each group would make their own plan. At the end of this meeting, the Roma student who in the very initial stages of the activity was negative towards the activity, approached the teacher to say that she would like to contribute and was provided with the students' names in her group. She probably found the pragmatically oriented character of the activity interesting and/or challenging for her.

3rd phase:Measuring the schoolyard's sides *(30 minutes)*

The main objective in this phase was for students to conduct the measurements of the sides of the school yard and to prepare a draft plan of it.

The first measurement happened to be launched by the above referred Roma student. Then one student of each one of the other groups measured the rest of the sides of the yard. A student from every group had the responsibility to keep notices of the measurements. For the measurement, the odometer was used; a tool that seemed to be very attractive for the students:

A student measuring the yard

4th session: designing the plan *(in the classroom, 2hours)*

The objective in this phase was for students to find the right scale, even though they would not express their results in a mathematical register. They also had to design the plan of the schoolyard on a paper.

The students worked in groups. Specifically, 5 groups were formed, 3 of which Roma students participated in. Classroom desks were arranged for students to sit around, thus facilitating the processes of collaboration.

<u>The students' difficulties in constructions and strategies:</u>

The initial strategy of all the students in order to depict the plan of the yard in their papers was to transfer meters to cm using in this way unconsciously the scale of 1:100. In some cases, when they realized that paper (A4 size) was not adequate, they changed the arrangement of length-width and marginally managed to draw the floor plan. After realizing that the paper was not enough, one group just enlarged the paper fixing one more sheet to their original one, while the other groups divided each size by two making the scale 1: 200. As we could notice, students did not immediately apply the rule of thirds to find the right dimensions for the plan on the paper but they used their informal strategies that led to a satisfactory depiction of the yard. However, using these strategies, apart from one group, the other groups were not able to express which the final scale they used was.

It is interesting to see the answers of the students regarding both the process and the final result:

> Student: *.....then, because it didnot fit we divided it by two.*
>
> Teacher: *only the big ones? Did you leave the small ones as they were?*
>
> Student: *In the beginning, yes, we did only the big ones, but then we thought to divide them all so that we get the correct shape.*

The students, in their justification, although they do not directly refer to scale, they suggest it, saying: "so that the shape would be right." They comprehend, that is, that the shape they have to make is the same as the shape of the schoolyard; they understand the concept of similarity and the notion of scale. The fact that it is an authentic activity dictates the need for a working solution in the particular context.

An additional difficulty that students faced in designing the floor plan of the schoolyard was that their final shape had to be a closed polygonal line. Having overlooked the need to measure the angles so that they had an exact mapping, the students were often led to arbitrary adjustments regarding some sides of the schoolyard so that they got a closed polygonal line.

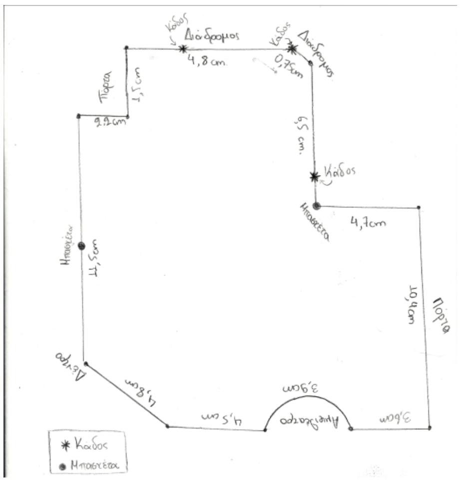
Teacher: *did your shape close properly?*

Student 1: *no, as it didnot close, we made the hallway a bit bigger.*

Student 2: *in order to close the design, we made 22, 4 cm more crooked, so that it could fit.*

Students working in groups in the classroom

Below, one of the designs (floor plans) that the students created in their groups is presented.

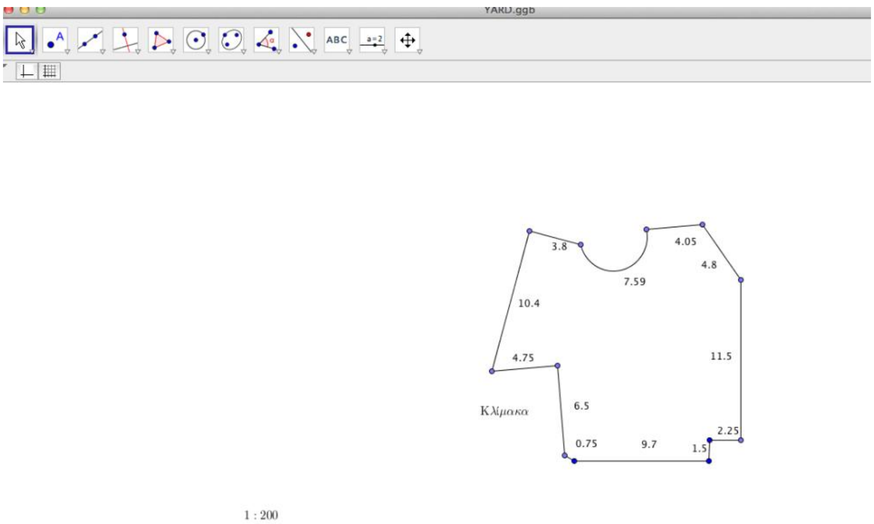


5th- 6th session: collaborative exploration of the groups' plans (1h+1h)

The teacher scanned all the groups' drawings and displayed them on a projector; each group had the chance to present the procedure of their design focusing on difficulties, strategies they had used etc. Furthermore, seeing their designs projected, students used the wireless mouse to describe and to show points that caused them difficulty as well as correct possible mistakes.

Then, the teacher introduced the GeoGebra displaying the screen on a projector so that all the students had access and so that after that they could make their own designs in their group, also showing and justifying the positions they selected for the bins to be placed at, with the use of icons. The teacher first constructed three consecutive rectilinear line segments as sides of the schoolyard. Since the shape of the yard was not a typical rectangle they faced the problem of making the figure a closed polygonal line. The need to measure angles surfaced through the teacher's suitable questions in order to determine the problem. This need was connected to the specific micro-framework, which is determined by the characteristics of the software and which had not appeared in the previous micro-framework -where the students worked with pencil, paper and geometry instruments- and in which they thought they could make arbitrary adjustments so that the polygonal line depicting the schoolyard closed. So, the teacher then, together with a few students, using the big protractor of the school, measured the non-right angles. After that, they put the values on their drawings so that they could construct the drawing more accurately with the software.

Finally, they jointly constructed the following drawing using GeoGebra:

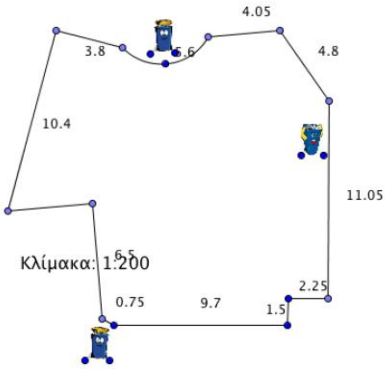


7th -8th session: the response letter to the municipality (1h+1h)

In these last sessions the teacher asked the students to justify the places they selected to put the bins in. Some characteristic answers of the students:

- the first bin should be in the amphitheater, in front of the column, because many children assemble there during the break;
- the second bin should be placed between the basketball hoop and the bike racks because many children gather there in order to keep an eye on their bikes;
- the third bin we think should be placed in front of the school canteen so that the students throw their rubbish there after buying food.

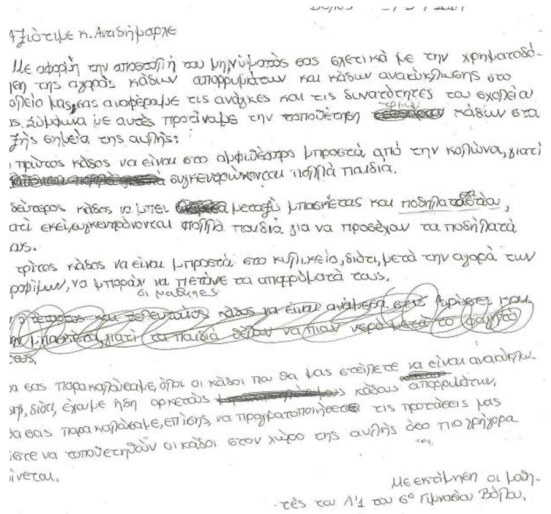
After plenary discussion for the best plan, the students resulted in one of 1:100 scale and they put the bins as it is shown in the drawing below:



It should be noted that the use of the software tool added a dynamic dimension to the drawing of the schoolyard and gave the students the possibility of experimenting in placing the bins, as well as showing the limitations of the previous framework, where the students drew the schoolyard using tactile materials.

In the end, the students wrote down an official letter for the administration of the municipality: dialogic construction of the text, teacher's scaffolding in the use of the mathematical register.

<u>Letter to municipality:</u>


Αξιότιμε κ. Αντιδήμαρχε

Με αφορμή την αποστολή του μηνύματός σας σχετικά με την χρηματοδό-
ση της αγοράς κάδων απορριμμάτων και κάδων ανακύκλωσης στο
ολείο μας, σας αναφέραμε τις ανάγκες και τις δυνατότητες του σχολεία
ς. Σύμφωνα με αυτές προτείναμε την τοποθέτηση τριών κάδων στα
ξής σημεία της αυλής:

πρώτος κάδος να είναι στο αμφιθέατρο μπροστά από την κολώνα, γιατί
συγκεντρώνονται πολλά παιδιά.

δεύτερος κάδος να μπει μεταξύ μπασκέτας και ποδηλατοστασίου,
ατί εκεί συγκεντρώνονται πολλά παιδιά για να προσέχουν τα ποδήλατά
ους.

τρίτος κάδος να είναι μπροστά στο κυλικείο, διότι, μετά την αγορά των
οφίμων, να μπορούν να πετάνε τα απορρίμματά τους.

να σας παρακαλέσαμε, όλοι οι κάδοι που θα μας στείλετε να είναι ανακύκλω-
ης, διότι, έχουμε ήδη αρκετούς κάδους απορριμάτων,
να σας παρακαλέσαμε, επίσης, να πραγματοποιηθούν τις προτάσεις μας
ώστε να τοποθετηθούν οι κάδοι στον χώρο της αυλής όσο πιο γρήγορα
γίνεται.

Με εκτίμηση οι μαθη-
τές του Α'1 του 6° Γυμνασίου Βόλου.

Conclusions

The teacher, who piloted the activity, as he participated in the workshops and had substantial contribution to its formulation, faced all the issues that appeared during the implementation of the activity creatively. The students, in turn, engaged with interest in a task-based activity, which regarded real problems in the school context and created conditions for an empirical approach to mathematics. The various micro-frameworks of cooperation among the students that developed gave the potential of reversing the traditional relations that prevailed in the class among students and between the students and the teacher. The students with a different linguistic and cultural origin (Roma) were dynamically involved in the cooperative performance of the mathematical procedure that their group had to perform; they made use of the linguistic experience of everyday speech in order to comprehend mathematical speech and negotiated mathematical concepts and techniques in a framework that was meaningful to them.

Finally, it has to be noted that although in the design of the activity the use of the language of home/community had been a possibility, the Roma students systematically avoided to use Romani, that is the language they used in their everyday transactions, in the class context. This choice of theirs is obviously connected to all the previous experience of those students and could not easily be changed in the context of only one activity.

Second piloting

by Maria Piccione**

TEACHING UNIT IMPLEMENTATION

The TU proposes an activity of *mapping* which typically involves basic geometrical concepts of school curriculum. It can be classified as modeling real space activities and then having the relevant feature to require the management of the relation *space size - sheet of paper size*. For this reason it represents a conceptual field that supports the development of methods and reasoning underlying geometrical competencies.

The activity is aimed to offer the students the opportunity to face a real life problem, without any indication of solution, exactly requiring tackling the construction of a *mathematical model*. By the given task, we expect they are able to identify and to enforce mathematical notions and techniques they already have (space notions, such as alignment, angle and length, shape, measure) and to get an approach to other ones not yet constructed (scale, proportion).

The activity provides a context for mathematical peers communication, which requires decision-making and negotiation of reasoning.

Description of the activity

This piloting took place in lower secondary school "Paolo Uccello", a school located in a neighborhood on the northwest suburbs of Florence, with a group of 7 pupils from a second class (degree) under the guidance of the trainer and the mathematics teacher (Prof. A. Scialpi).

The original proposal has been implemented with some changes. The first one

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concerns the choice of the space to be depicted, also to comply with school safety rules: a large hallway, well shaped for the relief work, inside the school, rather than the space surrounding the school - too wide on the whole and regular shaped in more limited areas. The conceptual domain remained unchanged, while the modeling work changed for what concerned certain realization features, being simplified. The environment taken into account could anyway be considered as a "*meso-space"* according to the ratio between physical space and subject perceptive space sizes.

The students were given the task to solve a practical issue by an official invitation letter on behalf of the School Director.

In addition, working with a group of students (and not with the whole class) other changes occurred.

The planned activity followed three phases within the times specified by side.

Phase 1(1 hour)

Presentation of the task. By reading the official letter, the students understood the request to collaborate in implementing a plan for the waste separation; for this task they had

- to make maps of certain building local places;
- to indicate appropriate settings for the rubbish bins and determining the number of these ones.

They also seemed to share the educative reason of their involvement aimed at developing a respectful attitude towards the environment.

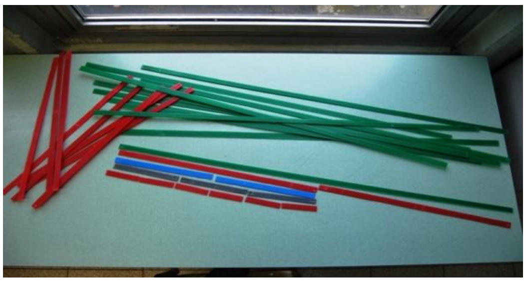
Work planning .The teacher did not explain the word map; she asked the small group of students how they would organize the work, by performing

- *what to do*
- *how to split tasks*
- *what tools they needed*
- *how to register the collected data*

It could be noticed that they easily realized they have:

- **to measure** the boundaries of the corridor taking into account the various openings (doors, stairs, windows) and obstacles (columns, radiators)
- **to adopt tools** to take measurements.

They were already familiar with a didactic material of rods and they spontaneously thought that the long rods were adequate to operate on big distances, while, on the smallest ones, the shorter rods could be used to complete the covering of the boundary lines of the floor.



Furthermore, they agreed they had:

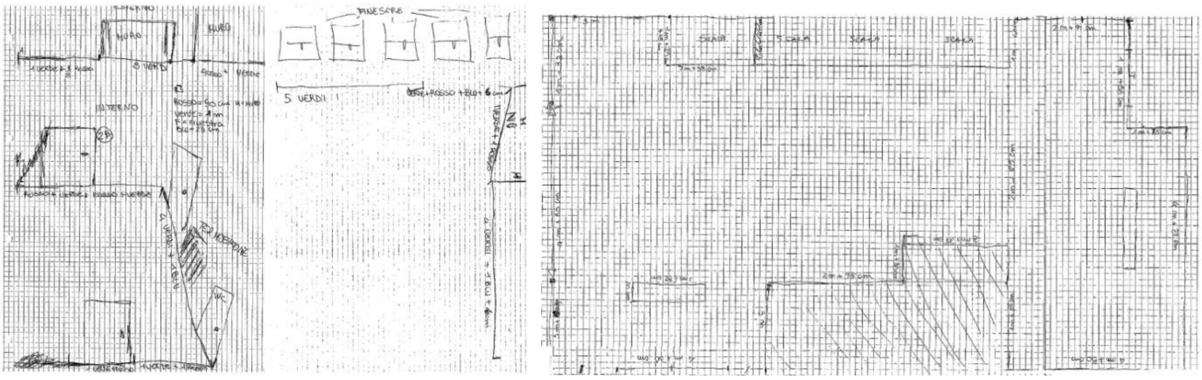
- **to record** the obtained measurements back on paper, and
- **to depictshape** to develop a model.

Common squared sheets of paper were taken for this purpose.

Phase 2 (2 hours in two times).

Taking the measurements. The students cooperated together at the new activity by placing the rods on the floor, along the walls, to take measurements; the need to split up some rods occurred to measure small remaining parts of the boundary. The trainer proposed to divide them into equal parts and the pupils agreed that this would have simplified the operation of setting length-relations between a single part and the one of origin. Then, 3/4 of them began mapping the results on the chosen sheets, without performing any choice of scale.

Developing the model. At the beginning, the "mappers" were just "drafters": they made a drawing taking in account the shape of single parts of the hallway, labeling the segment by the corresponding measures expressed in *rods-units* nominated by color. The first attempt of mapping was at a very elementary level with components of tridimensional representation.



The trainer suggested that the pupils carry out a global display instead of local ones: they stopped to analyze the whole structure and discussed about it.

Then she aroused the curiosity among the pupils to know the length of a long rod with respect to the standard units; and they made control: *just 1 meter*!

It is relevant to outline that the group depicted to the paper the representation of the hall, without considering at all a *scale-problem,* but only roughly respecting the ratio between object measures and corresponding segments' lengths measures.

In the end they recognized and agreed that they would have to decide in advance *how many squares* on the squared paper it was appropriate to let correspond to 1 meter. They became aware of the need of this convention and they would implement it for the final model.

It was clear that telling "square" they meant "sides of squares" but they found it natural to use more speedy expressions.

This shows the very didactical value of such an activity to approach the fundamental scale-concept itself.

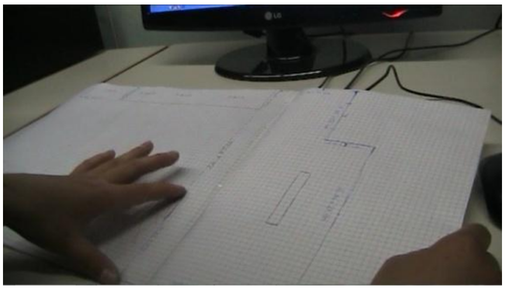
Various logistic comments and suggestions regarding the positions of bins were expressed during the activity.

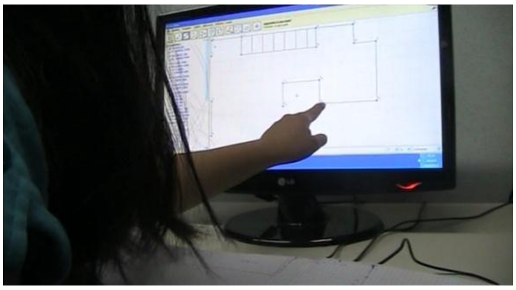
Phase 3(2 hours)**.**

Working in the computer laboratory. The students of the two classes had never used a software of dynamic geometry before. This situation put them in a new work context. They made use just of basic rules of GeoGebra, shown by the teacher, and were able to apply them to draw the official map.

As a first level of application, the work was made on the squared screen: this modality implies a reduced use of the software with respect to its real potentiality and aims, but at the same time it leaves the crucial problem to set a relation among lengths open.

They tried several choices and finally they found out an adequate correspondence leading to a ratio. Then a very careful transfer of data was made from the map drawn on the sheet to the rising one on the screen. By themselves they gave roles of "performers to the keyboard" and "prompters of data".)





Comments

***Affective domain*.**

The students took on the task with pride (having a social commitment) and seriousness.

While they were working in the corridor, curious people (students or adults) asked what they were doing: with enthusiasm they responded that they were carrying out a "relief" aimed at a School project, giving explanations.

The novelty of structure and typology of the activity (request, location, methods of implementation) let the students applying themselves to the work with interest, pleasure and sustained attention.

***Cognitive domain*.**

The activity offered a context in which the students had to apply mathematical concepts and procedures, revisiting them and increasing awareness of their theoretical and practical importance (measure as a procedure, units, unit conversion, results of a measurement, scales, proportions, model developing).

Itcreated opportunities for mathematical communication, which allowed pupils:

- comparison/discussion of ideas
- common decision making
- switching from common language to specific language
- verifying the difference of the formulation of verbal messages in both previous registers.

Difficulties.

The work gave the students the opportunity to face difficulties they did not expect in the execution of the model, mainly related to managing a non-standard shape and solving the problem of choice of the scale.

This aspect underlined a methodological problem: the teacher could realize the importance of setting up a problematic situation where the students find the conditions, have time and autonomy to outline the need of a new conceptual object.

Results.

Interview. An interview to the participants, a year after the activity, points out the presence of a clear trace of the work in long-term memory. Namely, the answers show that the activity allowed them to rethink

- the meaning of the measurement procedure;
- the role of the unit and sub-units choice;
- the effects of this choice.

We suppose that this awareness improvement is linkable just to the use of unofficial tools, what was non-planned a priori. Measuring with non-standard units seems to be a key-element that remained firmly acquired. All cite the measure with the rods as something that has affected them: "*the thing I was intrigued most was when we measured with the plastic bars"*; someone said that had always measured by the meter and were surprised to discover that it could be possible use rods: "*I never took measures by colored rods before that day"*. A girl speaks explicitly about the process of transition to the submultiples: "*and when we divided to measure the smaller parts"*.

Moreover, the answers outline that the learning context fostered the recognition of

- properties of the measurement procedure aimed at the realization of a map;
- actions ("*not simple*") necessary to transfer data collected from reality to the scale graphic representation.

The use of expressions such as: *"to draw back the perimeter of the area","to depict a map of the area"*, "*to measure the perimeter of a big area*" show that the relation boundary-interior of a figure is clear so as the operations to make, i.e. the meaning of the involved concepts exists but the corresponding terminology is improper and "fuzzy" and needs to be settled and well defined. The words "map" and "relief" should be emphasized, for it happened that they didn't become part of the language spontaneously (except in a case).

It is possible to notice that, the pupils recognized to be facing a true problem: transforming linear information (measures) in a two-dimensional shape. “*it seemed to be difficult, in the beginning, but we could speedily manage!*”

On the contrary, no reference has been made to geometrical properties they found in the space, in particular, the perpendicularity/parallelism between the walls and the rectangular shape of the columns in the hall: a posteriori, we relate this fact to the use of both the squared paper, and the grid on the screen of GeoGebra. We think that this choice facilitated the task to the point that the students were not aware of the whole system of underlying geometrical relations among the different elements of the figure. Undoubtedly this suggests the need of avoiding this facility and using a blank sheet of paper to bring out the problem during the relief/mapping activity and after using computer. Actually, under these conditions, the transition from paper to the computer screen seems to have been experienced only as a repeat of the same problem, only requiring more precision; except for the work necessary for the reproduction on scale that took place during this activity.

They also seemed to remember and pointed out the difficulties they met dealing with the particular space shape and wide size. This fact led us to make an experiment of visual exploration of a large double glass door standing to about 3 meters by it: they could express differences between managing the representation of a small or large space: the first one is easier because it allows “*to see the full form*”, “*to havefewer elements in the middle*”, *“to do fewer calculations”*, *“to spend less time”*, ...

Finally, on meta-cognitive point of view, other aspects emerged, mainly concerning the control of personal strategies of action (active listening of the teacher's advice and suggestions, being calm, confident, patient and precise, paying attention ...).

Third piloting

byPier Giuseppe Vilardo and Franco Favilli***

INTRODUCTION

In this activity students are expected to solve a practical problem through mathematical tools without knowing a well defined procedure. Definitely one of the key issues that will be introduced with this activity is *proportionality*, mainly through a geometrical approach.

The following task is requested to the students: they must identify the gathering points for classes outside of the school in case of evacuation; after identifying these points, they must provide unique written instructions to the municipality workers that will place poles in the collection points reported by students.

First, they will think over the task individually for about fifteen minutes, and then all the activities will be conducted in 6 groups of 3-4 students.

AIMS

Aims for students

General aims

- To connect mathematical notions and techniques to solve real social issues.
- To verbally communicate or by other means their ideas and their ways of mathematical action.
- To discuss their solutions.
- To be actively involved in exploring different exhibition paths for communication of mathematical procedures in different contexts.

Specific aims

- To measure.
- To use semantic structures to locate positions in space.
- To represent real objects in scale.
- To use proportions.
- To use geometric software to draw maps.
- To develop models.
- To use an appropriate mathematical language.
- To strengthen the knowledge of geometrical language.

THE CLASSROOM ACTIVITIES

Second Year students of Lower Secondary School; 12 activity hours (10 hours initially expected), 22 students.

The activity was carried out in a second class of the lower secondary school of the Istituto Comprensivo di Castelnuovo Magra (Province of La Spezia) and was used by

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the teacher as an introduction to the ratio, reduction scales and proportions.

Students were asked to identify, outside of the school parking lot (Photo 1), the gathering points for the 7 classes (out of 11 of the same school), coming out in the parking lot in case of evacuation. Earlier, another class had prepared the poles to be planted in the grass in the identified collecting points.

Photo 1: The school parking lot

After identifying the areas the students had to find a way to provide the municipality workers with understandable and unambiguous instructions so that workers could plant the poles exactly where the students had decided. This request was made orally by the teacher.

After making the request, the students were given about 15 minutes for individual thinking and then they were joined in 6 groups, (four groups of four students and two groups of three students).

Initially, the groups made plans and maps of the school and the parking lot; the maps were more or less similar to the actual situation of the school and the parking lot, and the students were concerned mainly with identifying the gathering points, basically according to their visual memories of the parking lot itself.

Then each group reported the solutions found. The groups were asked to report on the basis of their level of "progress", starting from "the most simple" solutions to the "most complex" ones.

The first group wrote verbal instructions in which they reported generically where the classes were to be located. Nevertheless, on the basis of these instructions it was impossible, for those in charge of placing the poles, to identify the right places.

Even the second group wrote verbal instructions, but they also added the distance from one pole to another.

The third group wrote verbal instructions, but less generic than the first two, because they used existing buildings (greenhouse) as landmarks and they also drew a simple map. However, they did not realize the importance of the map they had made, because they showed it only after the teacher asked them about it. Finally, after several questions, they realized that what they were doing was a map of the school, and that it would be very useful to indicate the position of the poles.

The following three groups all made a map: the map of the fourth group was the most simple, while, on the other hand, the maps of the fifth and the sixth groups were more elaborated. The fifth group also thought to measure the parking lot and to use these measurements to draw a map using a software that, according to the measurements, is able to draw maps. The fifth group pointed out that the map was the easiest and fastest way to solve the proposed problem.

After reviewing all the proposed solutions, the students decided that the map was the best solution. Then they started debating on which group had drawn the best map. The teacher then asked them what was necessary to draw a good map; all of them agreed that it was necessary to measure the parking utilizing suitable tools, such as 10-20 meters long measuring tapes. All in all, this activity lasted two hours.

During the following lesson the students measured the parking lot. Initially some of them were perplexed so they asked the teacher for help in order to start measuring accurately. A group utilized a laser pointer (they were asked how this tool worked; they knew how to use it but they did not know how it worked). After two hours all of them had completed the measurements.

After the measuring, the students started drawing the maps (they needed additionally two hours to complete this task). All the groups chose different ways to draw the school parking map. In particular, two groups completed their work at home, without being asked by the teacher, and in order to do so they used dynamic geometry software, such as GeoGgebra, and “planner 5d”, available on line; they had already thought about using this software. The other groups instead drew the map using graph paper. Only two out of six groups drew the map using a reduction scale.

When the maps were ready , all the groups showed and explained their works.

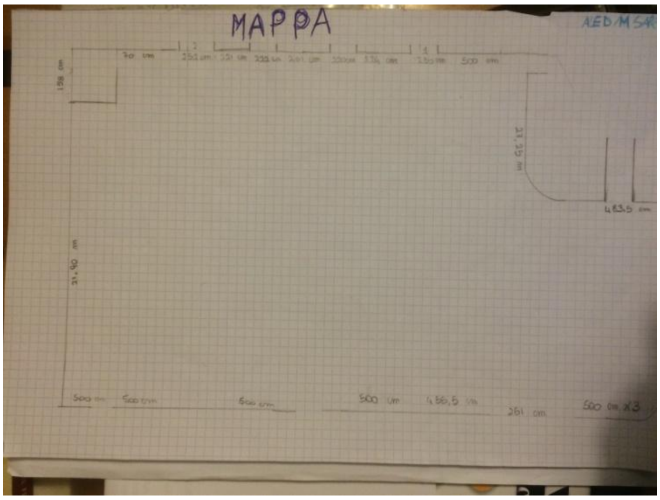

Photo 2: The map drawn by a group

From the explanations it was clear that four out of six maps were not useful because, when using them, it was impossible to allocate the poles in an univocal place (in two maps the poles were not even represented, but rather only the parking lot was drawn). When the students finished showing their works, there was a little discussion about what they did and most of them did not notice that four maps were not useful; so, many of the students' questions focused on issues of secondary relevance (in order to complete this activity we needed two hours).

For this reason the teacher decided to let the student verify on the field if the maps were correct (two hours activity). In order to do that he gave the poles to the students divided equally in the same six groups, and he asked each group to place the pole according to the map of another group. In this way they discovered that only 2 maps out of 6 allowed to place the poles (Photo 3).

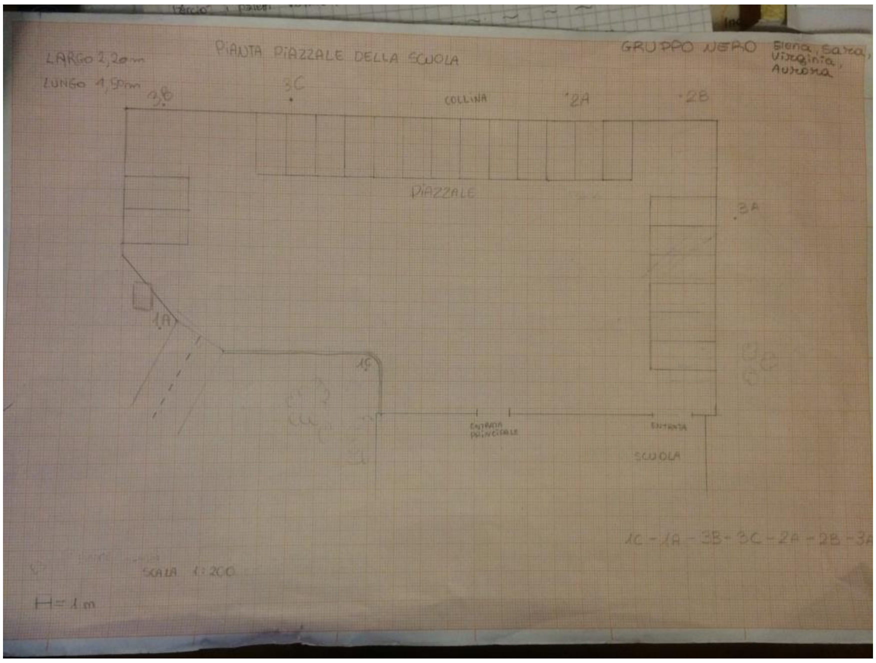

Photo 3: One of the maps with the poles location

In fact, some of them were not drawn with a reduction scale and others did not have the instructions about to place the poles, because the poles were not drawn in the map (Photo 4).

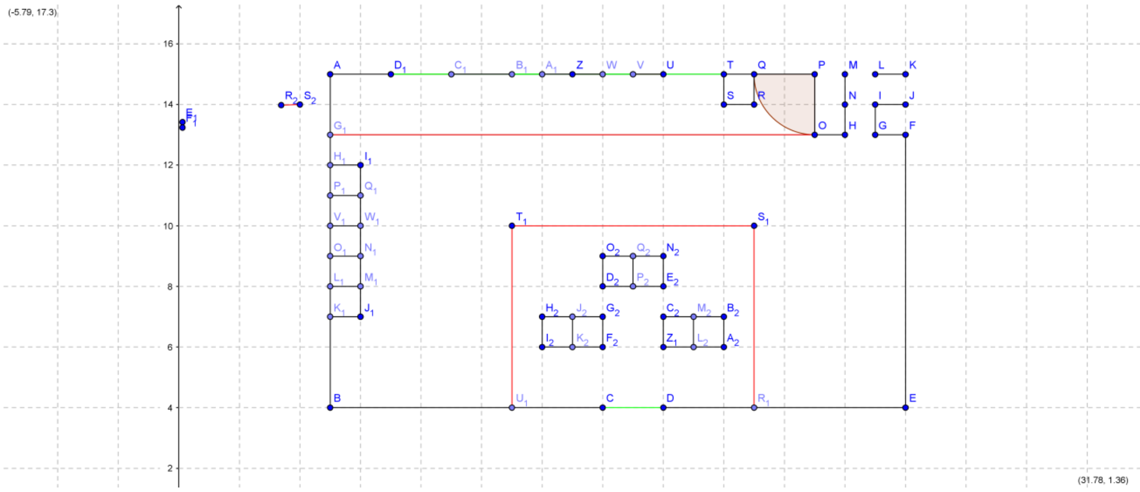

Photo 4: A GeoGebra map without the poles

So, the students decided that only 2 maps were correct. Finally, all the students wrote a report to describe what they had done.

CONCLUSION

First of all, we have to say that all the students participated enthusiastically and with great interest in the activity proposed, even if in most cases the results were not satisfactory, basically because most of the maps drawn (4 out of 6) were not suitable to answer the initial task. Nevertheless, all the students were able to recognize their mistakes and choose the correct maps. So, only in a few cases did the initial answer led to a spontaneous use of a graduation scale to draw the map in order to complete the task; however the work on the field showed the students the utility of such a mathematical instrument. In order to answer the initial question the students were free to explore their own path and only on rare occasions, especially during the talking among themselves, were they directed towards some kind of solution, such as drawing the map. However, also in this case they were not asked to draw the map in a particular way, but they were free to do it as they wanted. Thus, some of them utilized a line and a square and other geometrical software. Finally, they were not asked to make a scale reduction. So, the activity allowed to discover those students that had already used the proportions and that were able to use them correctly before they were formalized by the teacher. Obviously, only a few of them were able to do that. For the others, the activity was equally useful because it showed them how necessary this mathematical instrument was, thanks to the real-based task. Finally, in order to formalize the proportions, other real-based tasks were chosen to be done by these students, such as the preparations of recipes.

Fourth piloting

byAndreas Ulovec[****]and Therese Tomiska

The piloting

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the 6th grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team.

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Classroom piloting

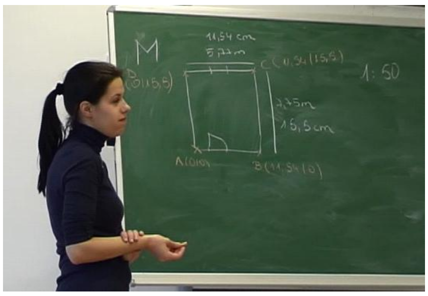
The teacher modified the teaching unit by not having bins situated in the school yard, but by having the bins situated in the music room during the school ball (which is where the buffet tables are situated during the ball) – a situation that was of interest for the students, since they will have to organize the ball and buffet in the next year. Also, the teaching unit was shortened to one lesson of 50 minutes. The lesson took place in the schools' computer laboratory, as suggested in the proposal.

The unit started with an introduction of the topic by the teacher (5 minutes) and continued with a brainstorming session about prerequisites. Very soon students realized that it would be convenient to have a map of the classroom to be able to try out the various situations and make an informed decision. The teacher had prepared such a map on paper and drew the schematics of it on the blackboard.

Why is this map not good enough?

This was followed by a session in which students discussed about whether more information on this schematic map would be needed. Students found out that without a scale it would be impossible to use this map for planning, and they selected a suitable scale.

Since the students were already in the computer laboratory, it was suggested that it would be much easier to have the map in an electronic form, so that it would be easier to try out several possible positions of the bins. Another discussion took place clarifying the prerequisites of transferring the map from the blackboard to the computer screen. The students found out that coordinates needed to be introduced and calculated, using the maps' scale.



Introduction of coordinates

After the basic room construction with GeoGebra, students found out that the size of the door was not obvious from the map. They started to measure the width of the door, using math text books as reference objects]. After this, the text book itself was measured with a ruler to convert the measurement results in metric units.

Measuring the door

Students then continued the discussion about reasonable positions for the bins and concluded that it would make sense to have the bins near the eating and buffet tables. They realized that they have to revise their strategy from positioning (small) bins to positioning (rather large) tables.

At the end of the session, students discussed what other constraints are given when they now need to position tables instead of bins, and weight, larger measures, as well as required minimum space between tables (for people to pass or stand and eat) were mentioned.

Conclusions

The piloting demonstrated that the objectives set in the proposal were definitely met in the implementation of this unit. The exploration of real life situations, the use of space notion, measurements, and scale, as well as the verbal communication on different levels all did occur.

Conclusions from the three piloting

by Charoula Stathopoulound Eleni Gana

The 4 piloting in 4 different schools (3 in lower secondary and one in upper secondary) showed that the activity was challenging for the students and they were actively involved in the process of teaching and learning.

The original design of the activity aimed to create rich opportunities of communication between students during searching for solutions and to facilitate students from different cultural and linguistic backgrounds to contribute through their own mathematical and discursive resources to the negotiation of mathematics notions and techniques.

A task-based activity was chosen, since: a) the pragmatic orientation of the activity was expected to cause the interest of the students, as mathematics corresponded to an empirical framework that was meaningful to them; b) the formulation of the problem did not provide predefined processes of solving the problem; c) it validated and built on the informal mathematical experiences and ways of expression that the students had (everyday language) to comprehend mathematical speech; d) allowed for various micro-frameworks of cooperation and communication among the students to be restored and for traditional relations among them to be undermined, mainly regarding students linguistically and culturally different.

In every school there was a different formulation of the problem, as the schools had a different formation of space but also different issues to address. However, the conceptual domain was the same as well as the mathematical content under question.

The fact that it was not a problem that demanded a simple algorithmic application but, on the contrary, the students were asked to choose strategies in order to solve it, activated the students to use exploratory processes and at the same time encouraged cooperation among them.

In all schools it was treated as a problem requiring modeling and the need to make decisions. Software of dynamic geometry were utilized so that they could enhance the exploratory dimension and the communication in the classroom through mathematics.

As was shown from the piloting, the students, having to face a problem in a realistic framework, activated self-regulation mechanisms when their original solutions were not effective. In addition, it was evident that the specific framework encouraged empirical and informal solutions and the teachers, through scaffolding processes, led the students to formal mathematical solutions.

Utilizing task based activities, we create opportunities of including students of different cultural and linguistic backgrounds and furthermore to challenge dichotomies that appear in mathematic teaching as, of:

- out of school and classroom mathematics,
- students that are listening and students that are doing,
- cognitive and emotional processing.

All in all, the activity proved challenging, interesting and meaningful to the students in all contexts, mobilizing them to address realistic problems in a creative way, while cooperating and utilizing everyday knowledge coming from different linguistic and cultural backgrounds to consolidate more formal mathematical concepts.

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