Multiculturalism, Migration, Mathematics Education and Language

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MASTERING MATHEMATICS, MAINSTREAM AND MINORITY LANGUAGES

by Franco Favilli*

INTRODUCTION

Little has been done in Europe as far as maths teaching in multicultural contexts is concerned. The different languages and cultures which may be present in the classroom make the teaching/learning process even more arduous than it already is, especially for pupils from minority cultures and/or with a migrant background or for Gipsy pupils.

A teaching unit is described below. Its aim is to provide teachers with a tool to help their pupils overcome the learning barrier represented by the contrast between the simplicity of classroom language and the complexity of mathematics language. Indeed, teachers have to bear in mind that the language used in class is an element of further complexity for pupils from minority cultures with a different native language.

The primary target group are mathematics teachers of primary and lower secondary schools in socio-culturally diverse areas, while the secondary target group consequently consists of students from cultural minorities and/or culturally deprived groups.

The educational aims

The educational aims of the teaching unit can be roughly divided into general and mathematical aims.

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Among the ***general aims*** the following may be considered:

- awareness of the positive values of cultures different from our own;
- creation of favourable conditions for intercultural dialogue in the classroom, and an inclusive educational setting by the use of different languages and pedagogical tools;
- development of awareness and critical attitudes towards the use of language and its interpretation;
- awareness of the role played by the use of a specific and unambiguous language in subject teaching;
- capacity to express the reason for the choices made and used during the activity;
- acknowledgment of the need to reflect on the texts and the role played by words;
- increase of students' ability to understand and process texts;
- deeper comprehension by foreign students of a written text;
- students' respect of the different working times of their classmates;
- fostering social relationships in the teamwork;
- development of students' autonomy.

Among the ***mathematical aims*** the following may be considered:

- increase of learners' capacity to understand and to elaborate the mathematical discourse;
- improvement of the ability of reading and understanding mathematics textbooks and word problems;
- improved usage of mathematical language;
- reinforcement of mathematical glossary knowledge;
- development of the ability to find a proper balance between natural language and mathematical language;
- identification of pre-knowledge and attitudes towards mathematics by foreign students.

The teaching unit should lead teachers to identify and reflect upon ***students***' ***potential needs***, such as:

- difficulty in using mathematical language correctly: uncertainties, doubts and mistakes shown in understanding the written texts express the need to favour the communicative process, when teaching, in order to help students to express themselves clearly and accurately;
- the need to use the linguistic competence appropriately since its use represents a fundamental step towards the construction of knowledge despite the time-consuming effect;
- the need to develop activities such as this one, because they offer information about pupils' knowledge, their conceptualisation level, their potential gaps, and misconceptions. This information is fundamental to be able to intervene in the classroom with appropriate and well planned teaching approaches.

Activities

The teaching unit consists of five main activities. All activities should be carried out in small groups, each of which including a minority pupil at least.

- **Analysis of a textbook** (*Reading* and *Writing*)

Students are asked to read a chapter of their textbook and then to search for and make a list of "difficult" words and verbs in the vehicular language, to discuss about their meaning and translate them into the foreign languages spoken in the classroom, thus producing a *micro-dictionary*.

Students are then asked to search for and make a list of words and verbs that are relevant to the mathematical language, compare them to the same words and verbs in the natural language, discuss about and write their potential different meanings and translate the words and verbs into the foreign languages spoken in the classroom, thus producing a *mathematics glossary* and a *mathematics dictionary*.

All groups are asked to re-write the analysed pages of the textbook in the vehicular language and minority pupils are asked to translate the most significant sentences into their own mother tongue language.

- **Analysis of a "word problem" from a National standard assessment test** (*Reading* and *Writing*)

The teacher chooses a "word problem" from a National standard assessment test that is meaningful as to the language used. Students are then given the same tasks as in the first activity.

- **Natural language and mathematics language**

Students are asked to identify potential conflicts originated by different meaning of words and verbs that are common to both the natural and mathematical languages, and to write the two different meanings in their own mother tongue language.

- **Writing a "word problem"**

Students, still working in groups, are asked to write a word problem in the vehicular language. The problems are presented to the whole class for discussion about the linguistic clarity and the mathematical notions required. Greater attention is paid to minority students.

- **"Writing a textbook"**

Students, still working in groups, are asked to write in the vehicular language a "page of a textbook" on a mathematical topic chosen by the teacher. The "pages" are presented to the whole class for discussion about their linguistic clarity and the mathematical notions involved. Greater attention is paid to minority students.

The main piloting

by Francesca Colzi, Stefania Massai and Franco Favilli

General information

School: Istituto Comprensivo "Don Lorenzo Milani" – Viareggio (Province of Lucca)
School level: Lower Secondary School
Number of teachers: 2
Number of classes: 6 (school year 2013/2014: two second and two third lower secondary school classes – school year 2014/2015: two second year classes)
Students age: 11 to 16.
Number of students: 63
Origin of foreign students: Albania, Georgia, India, Morocco, Romania, Russia.
Number of teachers in the classroom during the piloting: 1 or 2 (the subject teacher and the support teacher for special needs students).

Assignments for students

In order to foster a critical reading of the text, teachers can ask the students questions such as the following ones:

- *did you find it difficult to understand the meaning of some words?*
- *did you find most difficult the words in the common language or those of mathematical language?*
- *did you understand the overall meaning of the text?*
- *within the text did you get any link with your experience?*
- *does the text make you think of something meaningful?*
- *did the use of the Italian language dictionary help you?*
- *did the use of the language translator help you?*
- *what are the advantages / disadvantages of the use of a dictionary?*
- *what are the advantages / disadvantages of the use of a translator?*
- *did you already know the meaning of the words you searched on the translator in your native language?*
- *did the text written by your schoolmates make it easier for you to understand?*
- *how did you choose the words to use, when you rewrote the text?*
- *was it difficult to rewrite the text?*
- *did you find any words or phrases with ambiguous or unclear meaning?*
- *did the use of images facilitate your task?*
- *was it more difficult to rewrite the text taken from the textbook or to tackle the problem?*
- *how could the text be made more attractive and familiar?*
- *was it difficult to write the check test for your schoolmates*
- *were you able to carry out the check test prepared by your schoolmates?*

The piloting

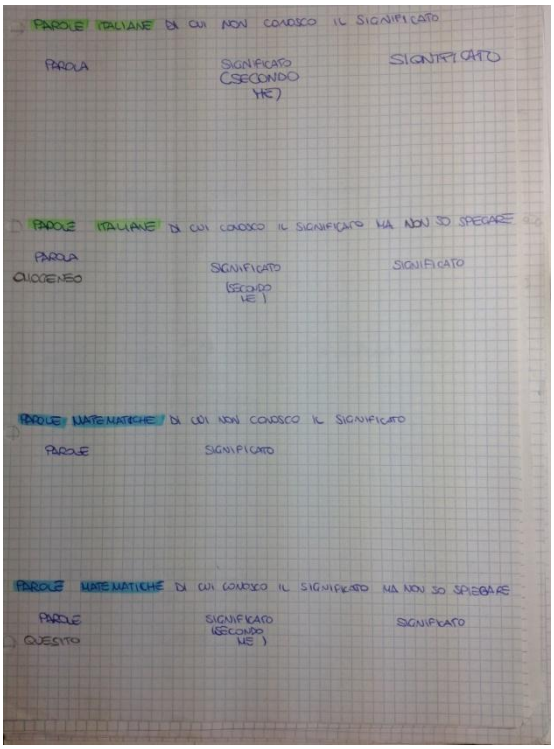
Activity 1 (school year 2013/2014 – grade 8 – 2 hours)

The chosen mathematical topic – probability – is not introduced yet.

The students' task is to read the pages about the topic in their textbook and to make lists of the words:

- they do not know the meaning of in the Italian language;
- they do not know the meaning of in the mathematical context;
- they know – but not clearly – the meaning of in the Italian language;
- they know – but not clearly – the mathematical meaning of.

After the reading, the students search for the meaning of unclear words in the Italian language.



Activity 2 (school year 2013/2014 – grade 7 – 3 hours: 2 hours by small groups of students; 1 hour by the whole class).

An "expert" group of 5/6 students read a topic about the "circle" in their textbook.

The group, purposely without foreign students, is asked to write a new text about the same topic so that it is clear to all their classmates. The group has also to prepare a test to check their classmates' understanding of the topic.

Activity 3 (school year 2014/2015 – grade 7 – 2 hours)

The activity refers to the teaching unit "Geometrical puzzles" in the LOSTT-IN-MATH project[1] (in which students are asked to work in pairs, one giving instruction

[1] http://losstt-in-math.dm.unipi.it/

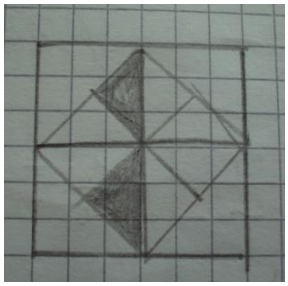
to the classmate who has to accordingly draw the geometrical figure the "instructor" has chosen without revealing its name.

Unlike in "Geometrical puzzles",

- students do not work in pairs, one student giving the rest of the class the instructions to draw – step by step, instruction by instruction – the figure he/she only knows;
- students are asked to draw compound geometrical figures and allowed to include the name of the "partial" geometrical figures in the instructions the "instructor" gives.

At the end of the instructions delivery, the resulting figures are compared and the given instructions are analysed and discussed to understand the potential reasons of the wrong drawings: ambiguous instruction or misunderstanding of its meaning?

Here below is the geometrical figure to be drawn:



Here following are the instructions given by the "instructor":

- *Draw a square!*
- *Halve the square by means of a vertical line!*
- *Draw a horizontal segment that starts from and ends to the half of the vertical sides through the centre!*
- *Connect all points you have found thus getting a rhombus!*
- *Split in two halves all small triangles you have got in the rhombus!*
- *Paint the top left and the lower left parts of the rhombus.*

An activity similar to the previous one was developed in another grade 7 class of the same school. In this case students tackle for the first time such an activity and the teacher decides to choose to describe an elementary figure: the rhombus.

The "instructor's" assignment is to give only "minimal instructions" (e.g.: *draw a segment, mark a point* …) without any further explanations to the schoolmates.

Here following are the instructions given by the "instructor":

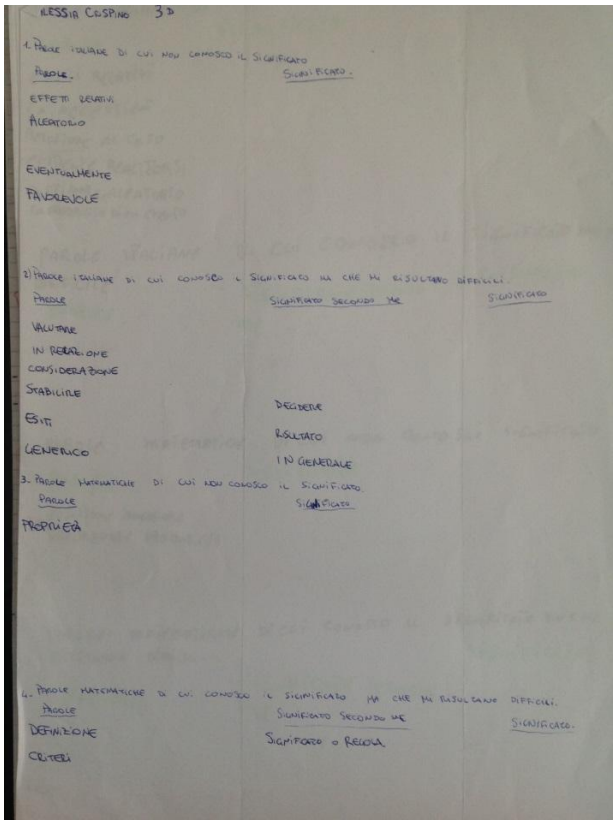
- *There is a line, yes, … a segment.*
- *With this segment draw an acute angle, …, upward.*
- *Then draw another segment to get an obtuse angle.*
- *Then, attached to this, another segment to get an acute angle.*
- *Then, attached again, another segment to get an obtuse angle that is equal to the first one.*

During the various steps, the classmates attempt to correct the instructor, trying to adjust and correct his/her instructions.

A posteriori analysis

Activity 1

The assignment is clear to all students, but sometimes they struggle to distinguish between Italian words and mathematical terms. Actually, none of them scans the text, so that they put in the list of unknown words even the vocabulary terms that cannot affect the overall meaning of the text. It may be that they are mislead by the assignment that requires to read the text and identify the unknown words.



The availability of the Italian dictionary proves useful when it is used for a few words; when the number of words to be searched increases , the reading of the text becomes much harder. Moreover, it is to be noted that dictionaries (both on paper and on the network) are mostly addressed to adult users and however, the lexical explanation is often difficult for young learners who therefore need the teacher's mediation.

A similar argument can be made for the use of a dictionary or online translator that appears to be useful only when searching for a single word, but not as effective for

whole sentences. The translation in the student's native language is useful in case of words drawn from the natural language, but it is not always the same when the words are drawn from the mathematical language and their meaning is unknown in the language of origin.

All in all, the proposed text on probability does not turn out to be easily understandable and does not allow students to build stable concepts.

Activity 2

In rewriting the text, students move away just a little from the original text, mainly doing a summary. At the end of the activity, the "expert group" says that the task was difficult because of the short time available, as they had to devote considerable time to tidy up the text which, according to them, was confused.

It is noted that, in the rewriting, initially, the mathematical figures were not deemed important, and therefore, were not retrieved from the textbook, although it is a geometry text. The students have put the figures only at the end of the text they have written, making links to them, as if the image interspersed in the text would make reading more difficult.

The rewritten text is understandable, but in some cases an oral explanation by the group of "expert students" is necessary.

The group was in trouble when preparing the check test, because they found it difficult to assess their peers and to design the test. One of the two groups preferred not to write the check test, while the other one initially had some resistance by peers who did not accept to be checked by the peers in the "expert group".

Activity 3

The activity elicited listening skills and attention both from the students who had to draw and the one providing the instructions. It also highlighted the students' difficulty to use the specific subject language. However, no particular difficulties were observed in the foreign students (all second generation immigrants).

As far as both the instructor and the rest of the class were concerned, this first experience pointed out very clearly, the ambiguities that may be originated by inaccurate instructions. Indeed, it was very difficult for the' "instructor" to find a logical order when giving the instructions to draw the desired figure. The difficulty to use a more rigorous and unambiguous linguistic repertoire than the natural language produced slowdowns and continuous adjustments.

In the end, the figures produced by the students were very different and distant from the expected result. This fact has stimulated a plenary discussion based on the following questions: "How do you explain that not all of you have drawn the same figure?"; "Why was the figure that the instructor wanted you to draw properly drawn only in a very limited number of cases?".

The students quite consciously identified the cause of the failure both in their weak mastering of the language and in the weak sequence of the instructor's commands to draw the figure.

Foreign students did not experience particular difficulties during the activity development, even those who speak a language other than Italian at home, because they all attended primary school in Italy and had therefore already approached the study of mathematics in this country from the very beginning of their schooling.

During the activity, the teacher's role was that of a facilitator that limits the most of his/her speech in the discussion and elicits the concepts as independently as possible within the classroom.

Second piloting

by M.-H. Le Yaouanq[**] and B. Marin[**]

TRAINING CONTEXT

As far as writing is concerned, one of the objectives set by the French lower secondary school curriculum for mathematics is "to train students to read and understand a mathematic text better, but also to produce texts which are then subject to a progressive improvement." Geometry can be seen as afavourable field for different levels of works in either enunciation, understanding of structured texts or in argumentation and validation. The official documents put the emphasis on making the difference between time for research and reasoning, and time for organising and writing this reasoning. They insist on not imposing too early a strict framework or a writing model: "The liberty of writing is one of the main driving forces for learning how to write. This also applies to proving". (Resource-kit "Raisonnement et démonstration pour le collège"[2]). As for competency-based assessment of the students, it is clearly asked to separate these two elements and examples are given to the trainees to be discussed.

Every year, many sessions in the trainees' education are dedicated to geometry in lower secondary school as it is an important aspect of the curriculum and the students' difficulties, as well as the teachers', are numerous when introducing deductive geometry and thinking organizing. Training is based on various theoretical frameworks like the different geometrical paradigms (Houdement and Kuzniak), the changes of registers of representation (Duval), and the tool-object dialectic (Douady).

BUILDING OF THE SEQUENCE

Lower secondary school curriculum offers very precise guidelines:

-"Attention should be paid to the language and to the various meanings of a single word".

- An "efficient way to convince students of the need for an accurate language is the shift from "doing" to "having something done". However, this requirement must not

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[2]Titre du document officiel
media.education.gouv.fr/file/Programmes/17/7/doc_acc_clg_raisonnement&demonstration_109177.pdf

look arbitrary to them. This is when the students have to write instructions to make someone execute them (for example, depicting a complex geometric shape so that it can be represented) or when they have to use a computer for a specific treatment that the need for accuracy appears necessary.

A first lesson will be dedicated to making the students aware of the polysemy of some mathematical terms.

The rest of the lesson will deal with what is called "telephoned figures" (the transmitter dictates instructions to a receiver in order to make him draw a certain geometrical figure) and on the use of a geometry software. Various experiments showed that students could efficiently communicate in *their* language by writing texts which are neither satisfying on the mathematical ground, nor from a language point of view, but they enable the receiver to execute the expected figure. The realization of the expected figure implicitly confirms the transmitter's text. This may question the interest of this work as an isolated work only aiming at working on language accuracy. However, many competences are at stake in this kind of activity: analyzing a figure, distinguishing the figure depiction from its characteristic properties, being able to change the registers of representation, and deconstructing speech into elementary instructions in a coherent and methodical way.

The choice is thus made to implement such a sequence on the «telephoned figures», but focusing on enabling the students to progressively improve their texts. Three sessions were determined in this way:

- a session dedicated to the production and the reception of the telephoned figures. The confrontation between the achieved and the initial figure should lead to an analysis of the problems that might have been encountered, such as missing information or information given in an inadequate order. The absence of language accuracy will probably not systematically be an impediment to the execution of a figure;
- a session designed for executing figures with a geometry software. The algorithmic process needs a list of primary instructions prepared by the work of a previous session. It can reveal potential defects, or implicit elements in the produced text, which would not have been noticed during the manual execution of the figure. The dynamic characteristic of the software also enables to highlight the work on the properties of the figure and not only on the drawing as the figure needs to resist to displacement;
- a last session aiming at improving the first produced text. A work of comparative analysis between "expert texts" extracted from textbooks and the students' texts should allow the students to identify some differences and then to rewrite their own texts.

This sequence takes place at the end of the school year in a class of year 7. Several allophone students newly arrived in France take part to it with the class while having an extra hour of specific tutoring with their mathematics teacher.

EXECUTION OF THE SEQUENCE

First session: polysemy of words

The first work relates to the "vertex". In theory, this term does not seem difficult; it is rather about making the work subject of the session clear.

Then, the term «altitude» is questioned. The notions of measure or size spontaneously come to mind in daily life. The mathematical definition is not mastered and the altitude is confused with the bisection or the median. The students all agree when the teacher asks if an altitude, in mathematics, could be measured: it can't be since it is a straight line. The calculation formula for the area of a triangle obliges them to reconsider their opinions and to notice the polysemy of the word in mathematics by themselves.

The session ends with a game available online[3] which requires matching a real life definition with a mathematical definition, both referring to the same word. The students autonomously think on paper before a collective summary shall be made. Participation is important: many proposals are made and are orally defended before checking on the computer with video projection.

The allophone students have been involved in the first part of the session; however the second part was too intense for them. The teacher will go back on some words during the hour of specific tutoring in order to start a glossary.

Second session: «Telephoned figures»

The instructions to execute the work on the telephoned figures have been practiced beforehand with the allophone students during their tutoring session. The suggested figures are simple and related to the Year 7 curriculum when the parallelogram, the characteristic properties of the square, the rectangle and the rhombus are taught.

The students work in pairs and the teacher dissociates the work by mainly proposing the square as a first figure, the rhombus is only given to four pairs. Two students who very recently arrived to France have to caption a construction video by choosing the terms from a list they were provided with.

Each transmitter receives the name of a figure and gives the instructions to the receiver who writes them down, draws the figure and mentions his doubts, or the pieces information that seem to be missing. Then the students confront the figure they had to describe and the figure they produced. If the square always leads to a square or to a rectangle by lack of mention of the lengths, the rhombus produces a varied range of figures. Only one group ends up with a correct construction program for the rhombus, starting with its diagonals. All the attempts relying on the sides from the other groups lead to wrong figures, and even to unexpected ones such as a hexagon or a triangle attached to a rectangle.

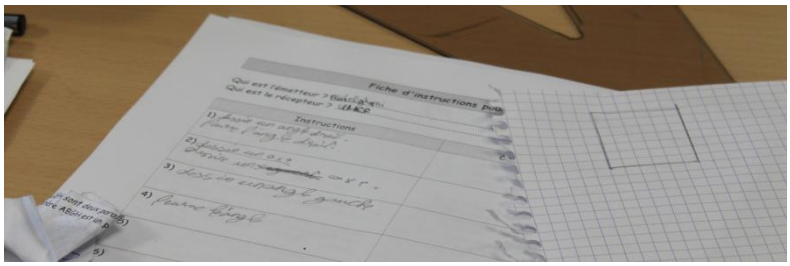
[3] http://matoumatheux.ac-rennes.fr/tous/vocabulaire/mots2.htm, available address on 2015 may 01

In most of the writings, the vocabulary which is used is not the math vocabulary. For instance, “draw a line” replaces “draw a line segment”. Three groups only spontaneously use letters to name points or segment lines.

Many students rely on spatial references, and the terms “horizontal” and “vertical” are often used to obtain perpendicular lines.

Thus, the instructions “draw a horizontal line/draw a vertical line/draw a horizontal line/close it” leads the receiver to the production of a square! The fact that the figure was given to the transmitter by its name and not by a drawing must have given an important indication to the receiver who executed one of the figures that he knows.

The same applies to the list of instructions given by an allophone student: “draw a right angle/draw an axis/draw a *left* angle/close the angle”, with which its neighbor produces a square.



One can question the meaning of the “left angle”: did the polysemy of the term “right” represent a problem for the allophone student? Is the left angle a right angle to be put on the left-hand side of the drawing? Yet, this student is aware that every angle is not a right angle.

Once work is done, the transmitter and the receiver swap roles and another figure is proposed, but many pairs will not have enough time to do it entirely.

A collective summary is carried out on the case of the rhombus, based on a wrong construction dictated by a group and drawn by the teacher on the blackboard; the word «height» is not used to describe the construction of a height and, when the teacher suggests it, the group does not adopt it. The students who drew a rhombus starting from the diagonals then propose their construction.

Third session: construction with a dynamic geometry software

The students work in pairs on the software GeoGebra and have to create first a square, then a rhombus and eventually a parallelogram. An extra figure is provided for the fastest groups and two groups will be able to deal with it. The students have to write down the functionality of the software they use. All the students draw the square. Some use the “bisection” or “reflection symmetry” tools to create the rhombus faster, making use of the summary of the previous session and to draw the diagonal lines first. The others take a long time to obtain a rhombus, as several constraints had to be taken into account for the same object.

Most of the students declare that the last session helped them do this work. By contrast, some allophone students who kept French as a work language for the software suggest that it would have been easier for them to start working with the

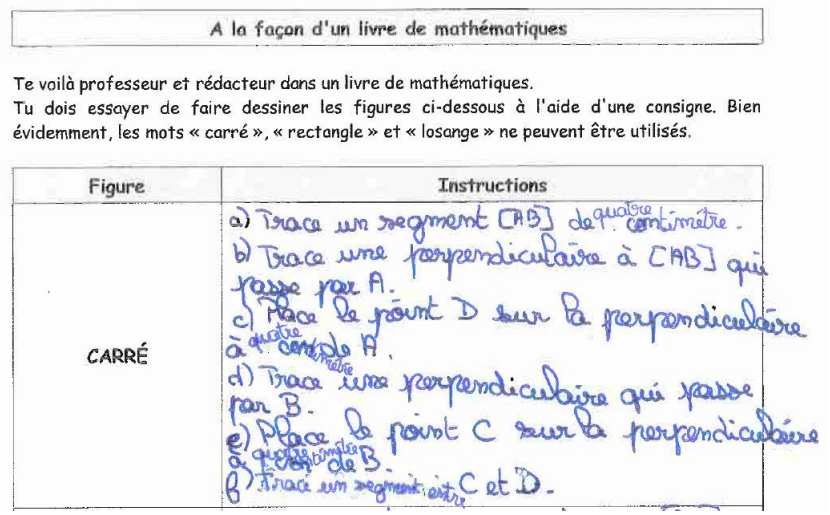
software in order to identify the words used and to write the instructions afterwards. The visual aspect of the icons as well as the feedback of the software enable the students to produce a figure autonomously and to learn or find the corresponding mathematical vocabulary.

Fourth session: Improving the texts

The French teacher of the class co-teaches this lesson.

After comparing their texts with an "expert text", the students in priority pick up verb forms, in the imperative, singular or plural, or the infinitive form. Other students focus on the vocabulary that is used and on the use of "mark" or "draw" in the expert texts by questioning the difference. Discussion follows in the class to explain this difference and to show the accuracy of the vocabulary.

The students rewrite their construction program.



A la façon d'un livre de mathématiques

Te voilà professeur et rédacteur dans un livre de mathématiques.
Tu dois essayer de faire dessiner les figures ci-dessous à l'aide d'une consigne. Bien évidemment, les mots « carré », « rectangle » et « losange » ne peuvent être utilisés.

Figure	Instructions
CARRÉ	a) Trace un segment [AB] de quatre centimètre. b) Trace une perpendiculaire à [AB] qui passe par A. c) Place le point D sur la perpendiculaire à quatre centimètre de A. d) Trace une perpendiculaire qui passe par B. e) Place le point C sur la perpendiculaire à quatre centimètre de B. f) Trace un segment entre C et D.

By trying to implement the produced program, the French teacher is used as a guinea pig by some groups both on the mathematical point of view and on the language point of view. Rich interplays allow further improvement.

POST-PROJECT ANALYSIS

"Some inaccuracies still remain but I think that there is a real progression compared to the first session when the writings on telephoned figures were poor", the math teacher declares at the end of this sequence.

The texts are still not perfect but the given pieces of information are more often exhaustive and do not refer to any spatial localization. They begin with clear orders ("Mark", "Draw",…) and the mathematical vocabulary is much more often used. The points and the segment lines are often named in order to simplify and clarify the instructions.

By confronting their texts to expert texts, the students found by themselves the shape that a construction program could have, without the teacher imposing on them an arbitrary and strict framework. Without any constraint, they could picture the type of text expected from them.

The work done on some terms can also help the students improve their understanding of the instructions in the future.

Finally, the requirement to transmit a text to another student or a software and the observation of the obtained result give meaning to this effort of writing and re-writing. However this work does take a long time; the figures have to be simple and the produced texts have to remain short. Yet, it seems essential, as most of the students do not discover by themselves the characteristics of such mathematical texts to re-use them autonomously afterwards. That is shown by the poverty of most of the first produced texts, in the end of Year 7. More sessions would obviously be needed to keep on developing and strengthening their skills to write a mathematical text so that they could be transferred to other productions, different from the construction program ones. This mastery of communication in mathematics also deals with mathematical knowledge and this writing and rewriting work allows working on this mathematical knowledge too.

References

Duval R. (2000), Écriture, raisonnement et découverte de la démonstration en mathématiques, *Recherche en Didactique des Mathématiques*, vol. 20, n° 2.

IREM de Strasbourg (2002), *Ressources pour le programme de sixième*, (brochure avec un CD contenant des activités directement utilisables pour les élèves).

Pluvinage F. (2000), Mathématiques et maîtrise de la langue, *Repères IREM*, n° 39.

Pudelko B. et Legros D. (2000), J'écris donc j'apprends, *Cahiers pédagogiques,* n° 388 – 389.

Third piloting

by Charoula Stathopoulou[***] Eleni Gana[***] and Ioannis Fovos

Introduction

The teaching intervention was implemented in two different educational contexts with corresponding student groups. The first group consisted of 20 students in the 1st class of the 6th Junior High School of Volos (aged 12-13 years old), seven out of whom were Roma people. The second group consisted of detainees (aged 17-21 years old) who attend the 2nd and the 3rd class of a Junior High School that operates inside a detention centre. The majority of the students in this group are students who come from Asian, African and European countries, have lived in Greece for a few to many years and can communicate very little to quite well in the Greek language. The teaching was implemented by the teacher of mathematics of the classes, Mr Ioannis Fovos. The aforementioned teacher taught mathematics in both schools. He had 25 years teaching experience in secondary education, and specifically 12 years of experience in teaching mathematics in a school context that functions inside prisons.

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In the second context, the piloting of the activities was implemented in cooperation with the teacher of Greek in the school, Mrs Anna Georgiou, who had previously been informed and had prepared for the specific teaching intervention.

Classroom piloting

1st group of piloting: 1st class of the 6th Junior High School

In the 1st teaching hour, the teacher informed the whole class that in the specific teaching context they would explore and reflect on mathematical language (discourse) and its relationship with the language we use in everyday interactions. Then the students were divided into groups of 3 or 4. They were given a worksheet with three activities and a page which contained parts of a text from the school mathematics textbook, from the unit "quadrilateral shapes" (parallelogram, rectangle, rhombus, square, trapezium, isosceles trapezium).

In the first activity, the students referred to their experiences and recalled words we meet in mathematics and our everyday life which have similar or different meanings in each context. All groups contributed with words/word meanings and the teacher wrote the words on the board. After that, mathematical words which are used in everyday life with the same or similar meaning were found in the text (2nd activity). In the *2nd hour of teaching*, the students worked on a word problem given to them and in the end they made one of their own, which they solved. Creating a problem triggered the students' interest, as it was the first time they had to put themselves into the position of making a problem instead of solving it. In the different context of mathematical involvement suggested to them, the students reacted with some insecurity and addressed their teacher more often in order for him to support or check their decisions. At the same time, however, there was more willingness for cooperation in the group and exchange of opinions on the way they would formulate the mathematical concepts and procedures into words.

2nd group of piloting: School inside a detention centre

In the school that functions inside the detention centre, the same steps and teaching tools were used in the implementation of the activity of word problems. There was especially great interest, however, on the part of the students-detainees for the communicative approach of the activity and the participation in the discussions both in the context of the whole class and in the groups was very high.

1st hour of teaching: Recalling words of mathematical language which are also used in everyday life (1st activity of the worksheet), as was expected, was difficult for students with limited resources in the Greek language. Associating the two fields of language use demanded abstract mental processes related to their absolutely framed experience of using Greek. The teacher of mathematics supported the procedure with guided questions and verbal hints and the teacher of language offered corresponding help going back to texts that had been studied in class. The second and the third part of the activity (recognizing words in a written text, association of meaning with everyday language and the equivalent in the students' mother tongue) supported a dynamic interaction among the members of the group. The students interacted in order

to locate the words and associate their meanings in the different contexts they are used. Students that had more resources in Greek acted as mediators, translating words of the text into their mother tongue, supporting in this way the access of the weaker students to mathematical language.

2nd hour of teaching: The whole of the class studied a word problem. The discursive traits of the word problem genre were recognized and modelled. After that, groups were formed by the students themselves, based on their common origin and language. Each group undertook to create its own word problem. The students negotiated among themselves not only the mathematical content of the problem they would create, but also the way they were going to phrase it, that is, the choice of mathematical vocabulary and the text organization of the word problem. The discussion in the groups often moved "between" two languages (Greek and the students' mother tongue) and was mainly about their effort to clarify mathematical concepts, but also to choose the appropriate expression in the context of mathematical language. The teacher, when asked to support the students, was informed about alternative suggestions and guided the students efficiently so that they would decide about the final word choice themselves.

Conclusions

The activity of word problems supported processes of conscious reflection on the relationship between mathematical language and everyday language for all the students, native and non-native speakers of Greek. Specifically, the students who were taught mathematics in a second language considered the activity very interesting and productive, because it allowed them to correlate and clarify the meanings the words took depending on the context in which they appear, namely in a mathematical or everyday communication context. Furthermore, those students pointed out that they had also tried on their own to comprehend mathematical concepts based on their first language, but it was not always easy. The validation of translanguaging in the classroom gave them the ability to understand more mathematical terms and concepts, although they had to rely only on the translation given by their classmates who spoke the same language, due to the limitations of the detention centre.[4] The teacher stated that in the future, when designing similar activities, he would provide the students with photocopies containing translations of the terms in their mother tongue.

In conclusion, both the teacher and the students found the construction of a word problem by the students themselves especially interesting and useful. The students pointed out how advantageous and stimulating it was for them to try to "transfer" an occasion and a mathematical problem to appropriate (mathematical) language. According to the teacher, the piloting of the activity was a good initiative for designing corresponding activities with word problems.

[4] With the activity as a starting point, the class talked about the limitations that the detention system imposed on students' online access to texts in their mother tongue and they wrote a petition asking to be granted permission to use the internet for educational purposes.

Conclusions from the three piloting

by Roberto Peroni[****]

Language is the most pervasive technological instrument for cultural communication and cognitive development in the human nature evolution, and almost each content and interaction need to be organized linguistically.

The relationship between Mathematics and Language is especially crucial because both every human language is "un système où tout (ou presque tout) se tient" (de Saussure and Meillet), and Maths is a strictly structured language not only with a dictionary of specific words but with a hierarchical syntax too.

The topic of the Project M3EaL is the relationship in the Maths classroom between Language and three M-factors: Mathematics, Migration, and Multiculturalism. A situational context where the relationship (the distance) is between ordinary language and mathematical discourse in a multicultural classroom, with various different native languages (Minorities' languages) vs. a second language which is the first language of the majority of pupils).

On detail, the three piloting reports present educational complex contexts with students with various migration backgrounds, who are native speakers of minority languages from East Europe, Africa, and Asia, - and in the Greek piloting activities pupils from Roma people too, and students (17-21 years old) inside a detention centre while attending the 2nd and the 3rd class of Junior High School. Everybody of them must "learn to learn in a second language (Gibbons, 1993); and if neuro-imaging researches have revealed a complex language and mathematics relationship showing left perisylvian language activities in exact calculation (Dehaene et al., 1999) and intraparietal involvements in approximation and quantity comparisons (Dehaene et al., 2004), a recent fMRI study (Wang et al., 2007) on Mandarin Chinese learners of English shows that, compared to L1, calculation in L2, but non parity, can be processed through the L1 system (language dependent) involving however additional neural activation, especially in the left hemisphere including the inferior frontal gyrus ((Broca's area).

On the teachers' side, job's aims are very high and complex: awareness of the positive values of different cultures, creation of conditions for intercultural dialogue in the classroom and an inclusive educational setting, development of critical attitude towards the use of language and its interpretation, and obviously teaching mathematics, i.e. to be able to increase learners' capacity to understand and elaborate the mathematical discourse, and to intervene in the classroom with appropriate and well planned teaching approaches.

The proposed and piloted activities are of explicit multicultural nature. Important common features in each piloting report are the analysis (reading and writing) of part

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of a textbook, and the analysis (reading and writing) of a “word problem” from a National Standard Assessment Test to search for difficult words meanings and/or difficult sentence meanings to reflect on, list them and translate into the different ordinary languages, and also for technical mathematical meaning. In particular in the French piloting report in order to avoid misunderstanding from polysemy: for example “vertex”, “height”, “right angle”.

Reflection activities on geometrical figures (“quadrilateral shapes”) are present in each of the three piloting reports, more effectively and pragmatically referred in the Italian (Pisa) and French piloting reports through a sort of telephoned figures sequences in order to try to obtain more accurate instructions, better mastering of language, and figure buildings more similar to the expected ones. In the Paris-Créteil report the students work also in pairs on the software GeoGebra to construct figures, with a little linguistic advantage in French language mastering (work language of the software) by allophone students.

It is a long time work after which some inaccuracies may remain, but it clearly represents a significant progress compared to the first session when the writings on telephoned figures were poor; the texts too may still be not perfect but they begin with clear orders (“Mark”, “Draw”, “Connect”, “Split” etc.), and the technical mathematical vocabulary is more often used, and the points and the segment lines are often named in order to simplify and clarify the instructions.

In all such activities the teacher is a facilitator, addressed by each pupils group both for mathematical and language issues: each further interplay allows further improvement, and it is clear that each step in the mastery of communication in mathematics deals with a proportional step in mathematical knowledge too.

To sum up, the experience from the three piloting shows that:

a) Translanguaging activities on word meaning problems, and on figures meaning support processes of conscious reflection on the relationship between mathematical language and everyday language for all the students, native and non-native speakers (of Italian, French or Greek respectively). Furthermore, the activities on lexical and word-in-text meanings have been considered more productive by students who were taught mathematics in the second language, because it allowed them to correlate the meanings of the words to the different contexts in which they occur: mathematical or ordinary everyday communication language.

b) The figure building activities support basic processes not totally conventionalised, connecting the more abstract symbolic level to the pragmatic movement and gesture on which abstract knowledge is grounded (Bates et al., 1979), (Arzarello et al., 2009), (Alibali et al., 2014), (Novack & Goldin-Meadow, 2015).

c) Translanguaging activities, well planned and led, allow better inclusion of students with different linguistic and cultural backgrounds, creating conditions

for intercultural dialogue in classroom, to be further developed out of the school.

References

Alibali, M.W., Nathan, M.J., Wolfgram, M.S., Breckinridge Church, R., Jacobs, S.A., Johnson Martinez, Ch. & Knuth, E.J. (2014). How Teachers Link Ideas in Mathematics Instruction Using Speech and Gesture: a Corpus Analysis**,** *Cognition and Instruction*, 32 (1), pp. 65-100

Arzarello, F., Di Paola, B., Robutti, O. & Sabena, C. (2009). Gestures as Semiotic Resources in the Mathematics Classroom, *Educational Studies in Mathematics*, 70 (2), pp. 97-109.

Bates, E., Benigni, L., Bretherton, I , Camaioni, L. & Volterra,V. (1979). The Emergence of Symbols: Cognition and Communication in Infancy. New York: Academic Press.

Dehaene, S., Spelke, E., Pinel, P, Stanescu, R. & Tsivkin, S. (1999). Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence, *Science*, 284, pp. 970-974.

Dehaene,S., Molko, N., Cohen, L. & Wilson, A.J (2004). Arithmetic and the Brain, *Current Opinion in Neurobiology*, 14, pp. 218-224.

Gibbons,P. (1993). Learning to Learn in a Second Language. Portsmouth, NH.: Heinemann.

Novack, M. and Goldin-Meadow, S. (2015). Learning from Gesture: How our Hands Change our Minds, *Educational Psychology Review*. New York: Springer.

Wang, Y., Lin, L., Kuhl, P. & Hirsch, J. (2007). Mathematical and Linguistic Processing Differs between Native and Second Languages: an fMRI Study, *Brain Imaging and Behavior* 1, pp. 68-82.