Multiculturalism, Migration, Mathematics Education and Language
Project Number: 526333-LLP-1-2012-1-IT-COMENIUS-CMP

A FACTORY OF TRIANGLES

by Maria Piccione*

INTRODUCTION

The proposal is related to one of the basic themes of geometrical knowledge, precisely it refers to the modality of conceptualization of the triangular shape, with its fundamental properties and relations.

The shape of geometrical figures is one of the cores founding of mathematical thought, and, together with the measure of extensions, it is a characterizing object of the Euclidean Geometry. In particular, the triangular shape is one of the most relevant official topics in Lower Secondary School, the main target of this proposal. The present work introduces the students into a proper field of study for the development of rational thinking, as Mathematics epistemology itself recognizes, and as the discipline teaching requires, in accordance with the statements of the Ministry Guidelines.

Aims

The educational aims of the proposal can be distinguished in cognitive, meta-cognitive and affective goals.

Cognitive aims:

* Department of Information Engineering and Mathematics, University of Siena, Italy

- Improving the visualization process
- Implementing the mathematization process
- Structuring the concept of constant/variable in a class of elements
- Developing graphical representation abilities
- Developing language, from common to disciplinary
- Constructing the concept of triangular shape
- Finding the existence condition ("triangular inequality") and the uniqueness conditions for triangles

Metacognitive aims:

- Developing awareness of the relation acting-understanding
- Developing awareness of the relation knowing-explaining

Affect aims:

- Creating sense of self-efficiency
- Arousing pleasure of doing/observing/discovering.

Outlines of the proposal method and design

Geometrical education has to refer and to emphasize the *subject-real space* and *subject-concrete objects* relations, in particular from the beginning to medium educational level. Points, segments, triangles and other entities - with which the discipline deals - are not concrete objects, but entities concretely representable or constructible by drawings or logically structured materials. A methodological perspective based on reality-interaction gives an efficient base for the intuitive stage of cognitive processes and theoretical applications. At present, progresses in neurosciences and cognitive sciences allow to assign scientific value to those statements which still were only intuitions, although experimentally proved.

Moreover, the *enactivism*[1], a general theory of knowledge connected to biology and to philosophies of experience, is influencing education by defining the *enacted model*, which focuses on *sense-making as a situated, embodied activity*. Castelnuovo's active learning method is consistent with this current framework and it is adopted in this proposal, involving part of its concrete material and manner of use.

The learning context is centered on the use of dynamical models , which are created by the elements of a simple material: a "geometrical Meccano" - consisting of rods, differing in colours and lengths (made of plastic, wood or metal), having holes at their ends or equally spaced holes along them - hooks and strings[2] (Photo 1).

[1]The theory, starting with the contribution of the biologists Maturana and Varela (1984), is explicitly introduced byVarela, Thompson and Rosch (1991).

[2] It would be better to allow pupils to produce the sticks by themselves, following written instructions.

Photo 1. The material for the activity

The work is intended to provide sensory-motor experiences of the object "triangle" and to manage language - in written or oral form - relating to the description of what is done and seen by pupils. Moreover, the topic enables a multicultural view about human knowledge and technology.

How the activity takes place:

- in pairs or in small groups of students, while building flat dynamical figures, analyzing products, drawing, writing observations or comments;
- in class for moments of discussion/comparison of the of results.

The proposal is divided into three sessions: Session 1, with its introductory function; Sessions 2 and 3 providing a natural cognitive path from the idea of three sided figure, to the concept of triangle, as far as to the congruence criteria. Each of them is structured in ordered steps.

A PRIORI ANALYSIS

The proposal is designed to face the present educational problems in a multicultural class, in particular:

- increase of differences among individuals (depending on cultures of origin, previous educational systems and school experiences, levels of relational and cognitive developments, levels of language knowledge as L2 ...)
- forms of unease (proved by behaviors such as isolation, mistrust or indifference to school activities and inability in adequate concentration/participation)

The seminal idea of the work is:

- setting an environment favoring action and individual experience in a social context, providing instruments apt to reduce verbal instructions.

The educational aim consist in:

- allowing everybody to implement cognitive processes, even in presence of linguistic difficulties
- anchoring the language development to experience and peer relations.

The choice of the material and its use aim, , at supporting the mental images generating process, necessary for the conceptual representation. As regards *image-schemata,* the great part of the work appeals to the *contact* schema.

THE PILOTING

This proposal has been piloted in two classes of the first year of the Lower Secondary School “Gandhi” (Firenze). The two samples respectively involved 22 pupils (6 migrant and 5 with learning problems) and 21 pupils (7 migrant and 5 with learning problems), two teachers (G. Sallustio and A. Scialpi) with the collaboration of the trainer.

It was first described by the trainer and discussed with the two teachers of the classrooms where it had to be piloted.

The work was carried out in three sessions.

Session 1: Becoming familiar with the material *(Duration:* 1 *hour and a half)*

Presentation of the “geometrical meccano”

The pupils of the two classes were gathered in a wide room. As the activity should have been performed with a new structured material, the first step consisted in its presentation and manner of use.

First free compositions

The activity took place in small groups of students (3-4), placed around school desks properly joined.

Each group of students was supplied with a set of sticks (15-20) and hooks: they created a lot of compositions, freely choosing and linking sticks together (figures 1.2, 1.3). Then they were asked to draw the nicest ones, and to reproduce the realized structures, on sheets of paper. These drawings were collected by the teacher at the end of the work.

Session 2: Setting up the motivating context *(Duration:* 4 *hours)*

Thinking/rethinking work*(30 minutes)*

This work was held in the theatre hall - provided with a projector and a screen - with the pupils of the two classes sitting in the platea area.

Firstly, every pupil was given three sticks and hooks for the conjunction and was invited to:

- observe the figure she/he composed;
- look into her/his minds – with her/his eyes closed – searching for images of triangular objects from the real world;
- write down the name and sketch the imagined object on a sheet of paper.

The teacher highlighted the various emerged examples of composed and imagined figures, and collected the sketched ones (Photo 2).

Commented projection*(30 minutes)*

An extensive selection of photos was projected on the screen, in order for everybody to become aware of the amount of triangles present in the real world. (Photo 3)

Subsequently, the activities took place in parallel in the two separated classes.

Photo 2

Photo 3

Internet browsing (*1 hour*)

In the computers' lab, pupils were divided in small groups and, partly guided by the trainer, were asked to search for websites on the internet concerning the triangles; they set the results of the search collecting the preferred representations.

A multicultural overview (*1 hour*)

The widespread use of the "triangulation" in any culture and time of the human history, from the earliest ages to the present days, was pointed out by the teacher by focusing on and by discussing some signifying examples:

- daily uses and applications (arrows' ends, tools …)
- first buildings (tents and bungalows)
- architectural structures (roofing technology, trellis building, network beams…)
- measurements
- mapping (buildings, flats, cities, lands …)
- works in arts and in architecture progressively advanced

The issue related to the successful reasons of the triangle naturally emerged; some of them were indicated as follows:

- stiffness of the triangular structure
- possibility to highlight and fix any point in the mapping process
- esthetic value

Two experiments*(1 hour)*

Two experiments were run in collaboration with the teacher of Technical Education:

- one aimed at checking the stiffness of the triangular structure, proving to compress real models, even the realizations made by means of folded paper;
- the other one consisted in making the classroom mapping by the use of distance-reliever. The room was approximately rectangular in shape and the students could understand and appreciate the necessity to draw a diagonal line in order to achieve the mapping. An assessment on the four sticks connected

procedure - by using the rods – clarified the situation and highlighted the sole configuration obtainable by fixing a stick of conjunction of two opposite vertices.

Session 3. Conceptual construction and language development *(Duration: 3 hours)*

The activity was organized in small groups of students (3-4), and was developed through different steps of increasing complexity.

Triangle existence conditions *(1 hour)*

The activity proceeded with the construction of a triangle joining three sticks. Each group of pupils was provided with two different set of triple sticks, in two consecutive times: the task consisted in combining the pieces into figures, in order to realize that:

1. the first triple leads to the production of *just one* "closed" figure (namely a triangle!)
2. the second triple leads to *a lot of* "open" figures …(Photo 4)

Graphical representations of the obtained figures in each situation - one pupil in the group may play the role of "reporter" - resolved to be a useful mean for registering the results.

Within each group, the teacher observed the *spontaneous* actions performed by the students during their activity, the verbal communication and the terminology in use (such as "rod" or "side", "open", "closed", "space", "proportion", "sum", "length"…). Progressively, the students of each group were asked to write down a comment on sheets of paper relatively to what they were seeing. The request was just "I see that …".The teacher collected them to analyze the resulting conclusions.

Afterwards, the teacher drew pupils' attention to the condition for triangle construction and to the number of possibilities, by posing the following problem (adapted from "Sticks and Triangles", 21° RMT, and referred to the name of a student):

<Ibrahim puts on his desk five sticks of different length: 4 units; 5 units; 6 units; 9 units; 11 units. How many different triangle could he build? Explain all possibilities.>

One given element (side/angle) triangles' construction *(1 hour)*

After suggesting the students to use sticks or other strings or elastic bands to make triangles, the teacher gave them time to work. Then, he gave rise a discussion within the groups by asking the following question:

- *What elements do you see that may change by movements?*

Two given elements (side&angle / side&side / angle&angle) triangles' construction*(1 hour)*

During this session, the teacher asked the students to set the material fixing one side and one angle and to use strings in order to extend one side and to close the triangle. They had to build, observe and make graphical reproductions of the obtained figures in various possible configurations. (Photo 5)

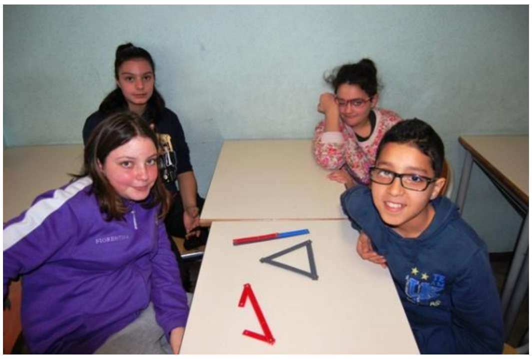


Photo 4

Photo 5

A problematic situation was created by asking the two following questions:

- *Does anything change if we provide another angle?*
- *Does anything change if we provide another side?*

As mentioned above, comments were gathered and organized.

Definitions

After this activity, pupils were engaged in a verbal description referred to some peculiar triangular shapes and in sharing the related technical terminology (such as isosceles, right, equilateral and scalene triangle). Moreover, pupils were asked again to imagine triangles belonging to the named classes and to describe the possible difference among elements of each class.

Problem solving

The problem solving phase was implemented to verify abilities in identifying triangles (see Appendix for the tasks).

A POSTERIORI ANALYSIS

Several results can be derived from the activity; we will focus on the most relevant ones, showing whether and how the implemented proposal accomplish its aims.

- Evidence of the potentialities of "geometrical meccano", within the mathematization process.

This practice has shown that the use of this material - in a socially shared context - favors the individual internal reconstruction of external operations. We will only discuss the role played by the material in discovering the "triangular inequality". The rod has shown to be an *artifact* which can evolve into the concept of "segment", represented by the linguistic *sign* "side". We could observe that, during practice, the three sticks do not play the same role. The pupils tend firstly to consider the rod with fixed ends - that they call "basis" - and therefore they take into account the other two rods, which have still some degrees of freedom - named as "residuals". The role of “residuals” appeared fundamental because it allowed to acknowledge the existence of a relationship among the elements on which the availability of the “closure of the figure” crucially depends. The following comments were obtained by the analysis of protocols related to the Session 2, in particular, by answering the effective request: “I see that …”. We could observe that this realization induces a spontaneous cognitive process, traceable by the words used by the pupils –such as *“short/medium/long”*, *“proportionate/un-proportionate”*, *“different/equal”* - and, as a consequence, by expressions referring to comparisons based on a metric interpretation. The fundamental steps of this process appeared to lay on two particular cases: the equilateral and degenerate triangles. The first case is characterized by sentences like *“it closes because the sides are equal”*. The second case – the unexpected one – reveals a powerful function, because it leads the pupils to the mathematization of the closure condition, in terms of “length” and “sum of lengths”.

- Instruments for the identification of logical/geometrical competences

For examples:

1. the inability to distinguish the two previous configurations in terms of quantifiers
2. the inability to deduce the corresponding congruence criterion from the possibility to construct a unique triangle - under some conditions.

- Instruments for the evaluation of cognitive discrepancies among pupils within the class

Examples of two borderline cases are:

1. the inability to represent graphically a certain configuration of rods
2. the ability to produce a perfect description of a combinatorial procedure (in the problem “Sticks and triangles”).

- Adequacy of the proposal to face the typical problems in a multicultural class.

Each student took part in the activity at least by touching and seeing real objects.

- Language development

The transformations of a concrete figure - by handling it - drive the subject to *say/try to say what he is doing* and hence helps language learning and put it through the hoops (presence/lack of words, proper/improper use, vague/rigorous use …).

- Influence on individual emotional components

The students were really intrigued by the activities.

They all enjoyed the work and could maintain attention for long times of application. The most striking case refers to a Chinese girl who had previously refused to participate in any school activity, and who finally began to become active.

- Impact on teachers training

At the moment, there are four teachers in the school sharing the will to revise mathematical contents and renewing teaching.

Suggested reading

Bartolini Bussi, M. G., Mariotti M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective, in English L. D. (ed.), *Handbook of International Research in Mathematics Education* (second edition) (746-783). Routledge.

Castelnuovo, E. (2008). *L'officina matematica: ragionare con i materiali*. Lorenzoni F. (ed.). Molfetta: La Meridiana.

Furinghetti, F., Menghini, M. (2014). The role of concrete materials in Emma Castelnuovo's view of mathematics teaching. *Educational Studies in Mathematics* **87** (1-6). Dordrecht: Springer.

*N*úñez, R. E., Edwards, L. D.,Matos, J. F. (1999). Embodied Cognition as Grounding for Situatedness and Context in Mathematics Education. *Educational Studies in Mathematics* **39**: (45-65). Dordrecht: Springer.

Rossi, P. G. (2011), Didattica enattiva. Complessità, teorie dell'azione, professionalità docente. Milano: Franco Angeli

Suggested links

http://www.di.ens.fr/~longo/geocogni.html

Second piloting

by H. Moraova**

Piloting / Siena – Triangles

Prague, May 27th 2014, 4 subsequent lessons, 7th grade, CLIL

The Czech team adapted the Italian teaching unit in such a way as to make it meaningful in the context of the school where the unit was piloted. It was piloted in a 7th grade of a lower secondary school ZŠ Fr. Plamínkové in Prague. As there are no migrant pupils in the class, the sequence of lessons was conducted in English language (CLIL – Content and language integrated learning) to simulate the environment where the learners are not fluent in the language of instruction. The sequence of lessons was conducted in one day.

Planning the teaching experiment

The teacher had to analyse the Italian teaching unit and adapt it to the conditions of a Czech school. The first obstacle the teacher encountered was that the construction kit used in Italy is not available in Czech schools. That is why she made the decision to use wooden straws instead.

Also a lot of attention had to be paid to introduction of terminology in the area of triangles in English (obtuse, acute triangles, right angles, equilateral, isosceles, scalene triangles, etc.).

** Faculty of Education - Charles University in Prague, Czech Republic

As the teaching experiment was organized in one day, it was also necessary to think of activities that would allow the pupils to have fun. That is why the teacher made the decision to allow pupils to use mobile phones to take pictures of triangles in the school building. This proved to be very motivating.

Course of the teaching experiment

1. *Activity one*: the pupils were given wooden skewers and asked to make sticks 4,5,6, 9 and 11 cm long, then they were asked to make as many triangles from these as possible (Photo 1). The goal of this activity was to make pupils formulate the principle of triangle inequality. The difficulty with using wooden skewers was that the end points were not pointed and thus some pupils managed to construct a triangle even in case that the sum of lengths of two of the skewers equalled the third one. This had to be explained by the teacher as the pupils were lead to formulation of erroneous conclusions.
2. Whole class formulates triangle inequality in English. The teachers checked comprehension.
3. Pupils were given a sheet with a number of exercises whose goal was to practice triangle inequality and make decisions about existence or non-existence of a triangle. This was the part with more calculations and less communication and reasoning, which is also very important in a situation when all pupils are not fluent in the language of instruction.
4. In the next part of the lesson, the pupils were introduced to terminology connected to basic properties of triangles (scalene, equilateral, isosceles, obtuse, acute, right). Using a powerpoint presentation the teacher first introduced the new terms and their illustrations on particular triangles. Then the teacher presented some pictures of triangles and elicited their classification from the class.

   Having finished this activity, the pupils were given sheets of papers and asked to cut out various triangles (Photo 2). The teacher then collected these triangles and distributed them to other pupils at random. They were then asked to take them one by one and write on them the right terms (e.g. acute, isosceles).

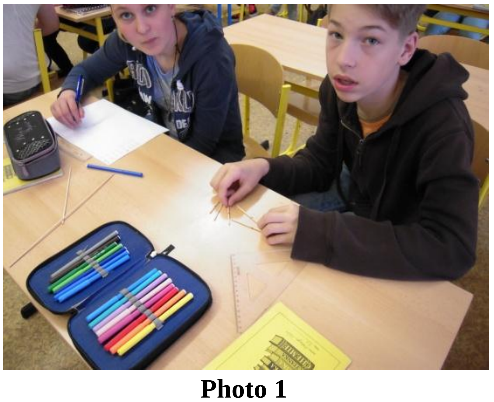


Photo 1

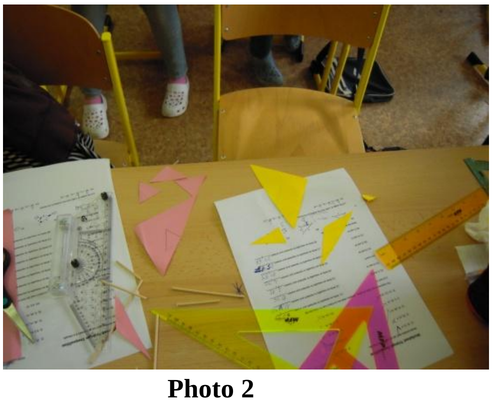


Photo 2

5. Having acquired the basic terminology, the teaching experiment could proceed to the cultural aspect of the teaching unit. First the pictures of triangles in

architecture provided by the project team in Siena were presented and the class discussed where they were used (e.g. Opera house in Sydney is world famous, Photo 3). Also use of triangles in various symbols was shown and discussed (Photo 4). This was followed by the part where the pupils were allowed to take their mobile phones, walk around the school building and take pictures of as many triangles as they can find. This part of the experiment turned out to be very creative and motivating. The pupils were more than willing to search and found triangles even in the most unexpected places (Photo 5 and 6). The pictures from the mobile phones were downloaded to the computer and presented using a projector so that the whole class could share. The pupils were asked to classify the various triangles using the acquired terminology. Also some misconceptions were uncovered (Photo 7 – the pupils made a photo of a pie chart claiming there were triangles despite the fact that one of the sides was an arc, not a straight line).

Photo 3

Photo 4

Photo 5

Photo 6

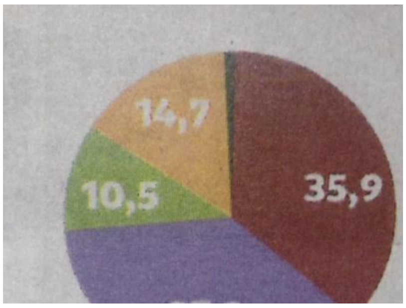


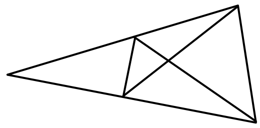
Photo 7

Photo 8

6. In the next part of the teaching experiment, the Italian teaching unit (see Annex) was used again and the pupils were asked to design a triangle tiled floor (Photo 8).

7. In the last part of the teaching experiment the focus was shifted back to culture. The activity started by the problem ***“Well hidden”*** *(question in 21° RMT)*

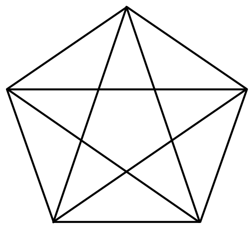
How many different triangles can you see in this figure?



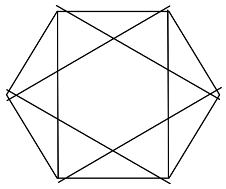
Then attention was moved to five point star, the six point star, the hexagram, Star of David and also to use of triangles on flags of different nations (search on the internet). The pupils started from the task from the Italian unit

How many triangles are there?

(a)



(b)



And then learned how to construct these stars, e.g. by drawing or folding a sheet of paper.

A posteriori analysis of the teaching experiment

The Italian teaching unit proved to be very nicely designed. It combines the traditional with the creative, mathematics with culture, drawing, calculating, manipulating objects, making conjectures and discovering principles. The pupils were motivated and active. They appreciated the use of mobile technology and the creative part of tiling their own floor.

Third piloting

by M.-H. Le Yaouanq*** and B.Marin***

INTRODUCTION

The third piloting was carried out by Mr. Paulou, a mathematics teacher at lower secondary school Roger Martin du Gard, a school located in an Educational Priority Area (in French: ZEP «Zone d'Éducation Prioritaire»), in Year 7.

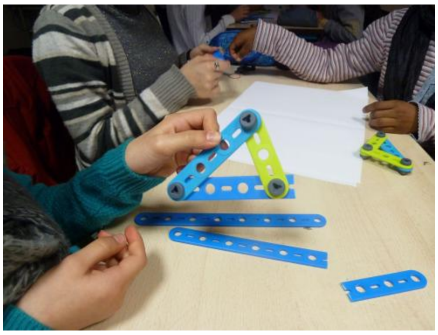
The notions attached to the triangle in the Year 7 curriculum are the triangular inequality and the triangle area. Cases of equal triangles do not appear in the curriculum. However we encourage the students to observe that in the case of a triangle construction, when one side of the triangle is drawn, one can construct several triangles, symmetrical two by two in relation to this side, in relation to bisection and to the middle of this side.

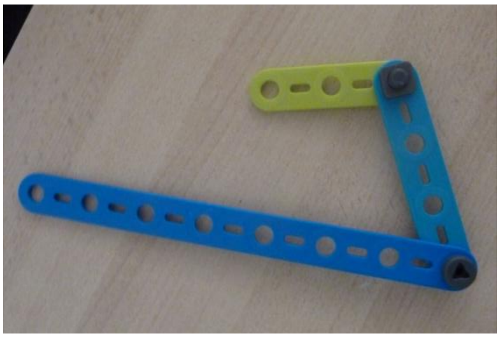
Two phases made up this course sequence.

PHASE 1: TRIANGULAR INEQUALITY

Three steps make up the exploration session.

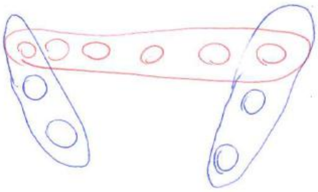
An action phase based on a game called «Meccano» in which plastic rods, with different lengths, can be put together with bolts. The students work in groups. Rods with different lengths are handed out and the students must try to build as many different triangles as they can.

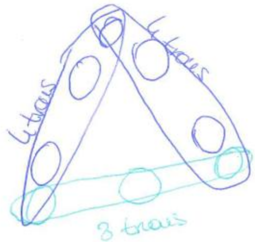




To keep a record of their trials, they have to draw the different results they have obtained on a piece of paper.

*** UPEC University, ESPE, Créteil (France).





Then a collective phase is carried out to conduct an inventory of the triangles that can be built or not. Then the teacher provides a game: the students must guess if someone can build or not a triangle with sides having given length. The teacher intervenes only to regulate, the students must debate orally. This phase shows how difficult it is for the students to find an exact formulation other than “gap not too big”, “close lengths”, etc. (Margolinas,2010).

On the other hand one can notice that, at the end of the session, the activity enabled all the students to develop a method allowing them to decide if a triangle with given lengths could be built or not.

The following sessions will be dedicated to formulate properties and to use them again, to reveal their existence and to show the different possible triangles during construction with instruments and their equality, based on the triangles built with Meccano.

We can notice that students often encountered difficulties with the case of the flattened triangle during construction with ruler and compass (with drawings of no flattened triangles with lengths sides 4, 5, 9), which did not appear during the activity, as the handling led directly in this case to the superposition of rods.

PHASE 2:

The students had to carry out a research phase about countries’ flags in which a triangle appears. They had to choose one, to describe it on a geometric point of view, to represent it and to look for the meaning of the different flag elements in relation to the country represented by this flag (shapes, colours).

This work had to be done in pairs outside school, then to be written and presented in class. The students contributed to this work with enthusiasm and gave back particularly careful productions.

This work was completed with area calculations in relation to the triangle area and the expression of proportion with percentages. Many possible openings like construction program writing or interdisciplinary work appeared during this phase.

Reference

Margolinas, 2010, Un point de vue didactique sur la place du langagier dans les pratiques d’enseignement des mathématiques

https://hal.archives-ouvertes.fr/halshs-00470238/document (retrieved on 01/05/2015)

Conclusions from the three piloting

by Maria Piccione

CONCLUSION

The whole experiment combines two components concerning *what* and *how* a certain topic could be conducted in a multicultural class: a teaching model, on the one hand, and a set of practical suggestions, on the other one. The obtained results encourage us to keep going in this work.

Some working choices and pupils' behaviors are to be pointed out from the three piloting, because of their mathematical value and positive feedbacks.

In the second piloting, the material was composed by simple wooden skewers: hence, it allowed the pupils to achieve a sensorial experience of triangular configurations, the basis for the next activity on triangular inequality. Moreover, the experiential training was supported by activities as paper cutting/folding, designing - even complex - "triangulated" figures and tiled floors. The work gives an example of how to reach higher levels of application, concerning the concepts of linear length and angular amplitude. The path can be followed through the questions posed in the worksheets: they require comparative operations and relations between linear length and angular amplitude, as well as the concept of variation range for lengths' values.

In the third piloting, the original Meccano was used. In this case, we find a further explicit reference about the effects of physical action on recognizing the constitutive elements of the model and their possible movements and dispositions. In accordance with the results of other studies, the material also reveals better potentialities than the rule-and-compass, for the understanding of geometrical properties, such as triangular inequality. This evidence can be explained taking into account touch and visual perceptions in a 3-dimensional environment. This benefit is clearly completely missing in a context where figures are already drawn, as the traditional teaching implies.

Useful and contextual suggestions also emerged from this piloting: the equidistant holes, following the length of the Meccano pieces, seem to induce the pupils to express the "closure of the triangle" in terms of holes' number (as a kind of measuring) (see Photo 2). Moreover, as regards the sense of visualization, a point of interest is represented by the discovery of *symmetry* in the distribution of the triangles, realizable under the condition "one fixed side", with respect to the side itself. Some steps towards measuring are done by computing areas and ratios of areas.

In each piloting, the research of triangular representations in the surrounding environment, that is in the natural, as well as in the cultural world, gave rise to a multicultural discussion. In particular, the explanation of triangle' meanings in cultures, and the research in countries' flags – in which a triangle appears – deserve to be emphasized.

In conclusion, by exploring the “wide and deep sea” of triangles, apart from the route, participation and careful production, we can state that interest and even enthusiasm aroused.

Further developments of the proposal

We can give some examples of natural developmental lines of the proposal:

- introducing the similarity criteria and approaching the geometrical concept of shape, differentiating its “gender-sense” and “specie-sense”
- extending the study to quadrilaterals and other polygons classes
- going to measure questions (perimeter/area)
- construction writing program

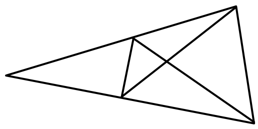
reproducing the proposal by ICTs use, such as GeoGebra , where “sticks” become “segments” in a 2-dimensional space.

APPENDIX

Problems

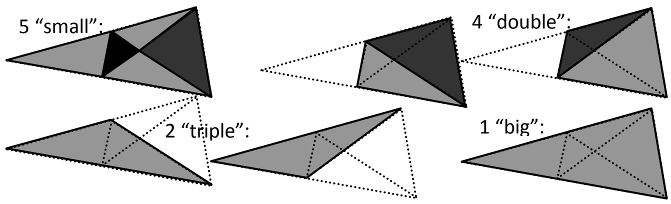
- ***“Well hidden”*** *(question in 21° RMT)*

How many different triangles can you see in this figure?



Involved processes: *visualization, recognition and counting*

Solution

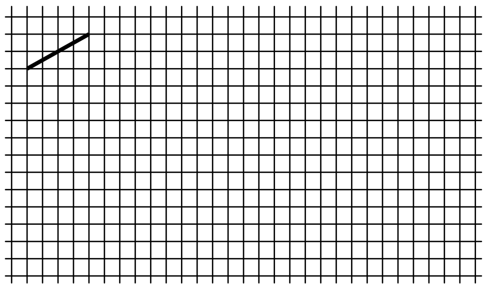


- ***“Triangles clipping”*** (problem in 19° RMT)

In this checkered paper find different triangles (i.e. not-overlapping) following these two instructions:

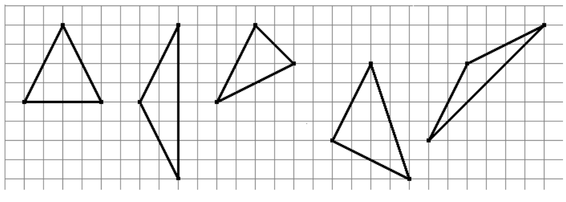
- *they must have two sides with same length of the segment drawn on the down below checker.*
- *they must have their vertexes in the intersection points of the checker.*

How many triangles you can find? Draw and then clip them.



Involved processes*: shape and measure identification; deduction*

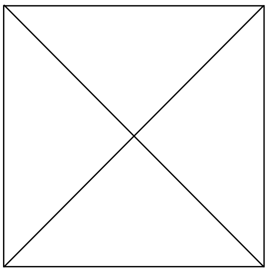
Solution



- ***"The composed figure"***

A figure is drawn on this paper. How many triangles with different shape can you see in it? Can you describe the shape you find?

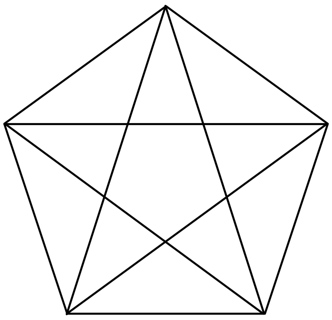
(a)



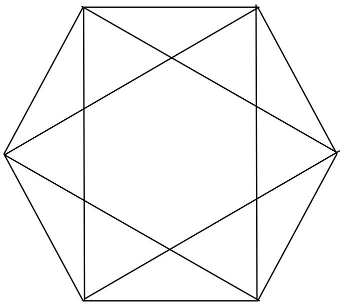
Involved processes: *shape analysis and classification*

Two more difficult version of the problem

(b)



(c)



- ***"A floor made of triangles"***

Create a tiled triangular floor, as you like it better.

Involved processes: *shape analysis and combination*