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Ready-made materials or teachers' flexibility? What do we need in culturally and linguistically heterogeneous mathematics classrooms?

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Abstract: The paper focuses on the issue of coping with the increasing cultural and linguistic heterogeneity in mathematics classrooms across Europe. The authors come out of a teaching unit developed by Barbro Grevholm within the project Multiculturalism, Migration, Mathematics Education and Language. The goal of the project was development of teaching units supporting linguistic and cultural diversity in mathematics classrooms. The authors of the paper argue that teachers of mathematics do not need detailed teaching units (although a survey among them shows this is what they are convinced they need as support) but they need topics with different cultural origins which they then adapt to suit the needs of the particular group of learners, their abilities, skills, age and language competence. In the presentation and the final paper the authors will show how the same teaching unit was grasped by different teachers and what the outcomes of their approach was.

Résumé: L'article concerne l'adaptation à l'hétérogénéité culturelle et linguistique de plus en plus présente dans les classes de mathématiques dans toute l'Europe. Les auteurs partent du concept de l'unité d'enseignement développé par Barbro Grevholm dans le projet «Multiculturalism, Migration, Mathematics Education and Language». L'objectif de ce projet est le développement des unités supportant la diversité linguistique et culturelle dans l'enseignement des mathématiques. Les auteurs affirment que les professeurs de mathématiques n'ont pas besoin d'unités détaillées (quoiqu'ils soient convaincus que c'est ce dont ils ont besoin pour les réaliser), mais que, par contre, ils ont besoin des thèmes provenant des cultures différentes qu'ils pourraient adapter pour les besoins de divers groupes des élèves, c'est-à-dire pour leurs capacités, leur âge et savoir-faire, pour leurs compétences linguistiques. Dans la présentation, tout comme dans la version finale de l'article, les auteurs montreront comment une unité d'enseignement a été modifiée par les différents professeurs et quels ont été les conséquences de leur procédé.

Introduction

The paper focuses on one of the much discussed issues in mathematics education, which is inclusion of multicultural elements into mathematics lessons. The paper presents partial results of the project 526333-LLP-1-2012-1-IT-COMENIUS-CMP Multiculturalism, Migration, Mathematics Education and Language (M^3EaL), whose main goal is to make teachers aware that pupils' culture (including their language) plays a significant role in the teaching/learning processes, also in mathematics. This awareness helps teachers to pay more attention to the different cultures (and languages) in the classroom and give them a higher value in the educational process. Conditions for an intercultural dialogue in the classroom as well as for better inclusion of pupils from different cultures are thus created.

The project aims to design and implement, in each of the partner countries, teaching materials for mathematics, which take into account situations or activities typical for specific (or even a variety of) cultural areas, as well as the role played by language in the teaching/learning of mathematics within multicultural and multilingual classes. The experimental implementation of these modules is

expected to lead to the identification of good practices to be exchanged inside and outside of the partnership. The proposed teaching materials for the maths classroom, inspired by practical problems and situations from everyday life and from different cultures, encourage teachers to take into consideration the different cultures in the classroom, highlighting their positive aspects and establishing intercultural dialogue in the classroom, thus promoting better inclusion of minority pupils.

The presented paper here discusses what form the developed teaching units should have – should they be developed with respect to a particular age group or school level, or should they only outline the possible multicultural content and leave it up to the teacher to elaborate it for the needs of their classroom and curricula?

Research in multicultural elements in mathematics education

It is generally accepted (e.g. Barton, Barwell and Setati, 2007; Bishop, 1988; César and Favilli, 2005) that mathematics teachers feel the necessity for training and materials which reflect the needs of their students in terms of linguistic and cultural differences. Their pupils from minority cultures and/or those with a migrant background encounter more difficulties than their native classmates in acquiring fundamental mathematics skills. Moll et al. (1992) claim that these different cultural backgrounds also provide "funds of knowledge" (i.e. "historically accumulated and culturally developed bodies of knowledge and skills") that can support the learning process. However, these "funds of knowledge" have only limited applications in many European classrooms, since they require a close connection and collaboration between teachers, parents and the minority community, and are mainly applicable if there is only one, fairly homogenous minority culture present, which is not the case in most European school backgrounds. Mostly however, learning a new language and culture at the same time as you learn mathematics places additional burden and challenges on migrant and minority pupils (Norén, 2010; Steinhardt & Ulovec, 2013).

Two years ago Ulovec et al. (2013) spoke in their paper of the lack of attention paid to multicultural aspects of teaching mathematics in contrast to a relatively large amount of research relating to multiculturalism in general without really making a distinction between subjects. Only some research focused on the difficulties in relation to the teaching of a particular subject and, moreover, it mainly covered teaching of language or natural sciences (McDermott & Varenne, 1995); research on mathematics teaching was rarer.

However, the changing reality in the classrooms across Europe started to attract attention of mathematics educators and now one can come across a variety of researches focusing directly on the specifics of teaching a culturally and linguistically heterogeneous group of learners in mathematics. A whole working group Multiculturalism and reality at the last CIEAEM conference in Lyon was trying to approach the issue from various perspectives (Aldon, Di Paola and Fazio, 2015). Attention was paid to the difficulties an individual with minority background faces in mathematics classrooms, in problem solving in exams, to the issue of what mathematics actually means to different groups of people and what value they attach to it, to the patterns of parental involvement in their child's mathematics learning in different sociocultural groups, to stereotypes in mathematics assignments and how these may affect minority pupils, to relations between experiences, languages, culture and power in multilingual mathematics classrooms and study the concepts of discourse and agency. Attention was also paid to the issue of how to make the mathematics curriculum more meaningful to minority (Roma) children and how a more meaningful curriculum could improve their participation and performance. This is in line with Meany and Lange (2013), who discuss the issue of learners' transition between contexts and warn of the additional difficulties for learners if their experience of home context is very different from contexts

they come across at school.

This shows that the number of perspectives is considerable, moving attention from one student and their background and obstacles to discourse and power in general, to meaning of mathematics to different learners, to cultural obstacles and to how to construct more meaningful curricula to allow learners from different sociocultural background to get involved. Santomé (2009) warns that if schools are to contribute to increasing justice and equity, they will have to analyze to what extent the curriculum is respectful of people's different cultures.

The authors of this paper are convinced that culturally heterogeneous learning environments in mathematics will allow learners to get acquainted with other cultures and their values. Moreover, they offer them novel, innovative ways of solving a problem, can offer new tools and procedures that are used in other countries and cultures and may develop their creativity and originality of methods used. Obviously, inclusion of elements from other cultures in mathematics will be of benefit both to majority and minority learners in the classroom.

A questionnaire survey conducted within the project M^3EaL showed that teachers feel an urgent lack of materials that they could easily use in heterogeneous classrooms. They ask for elaborated teaching units that would be ready for their use. This attitude is understandable, the workload teachers in countries across Europe face is enormous. However, taking account the variety across Europe, this means developing thousands of different teaching units.

The project partners agreed they would develop teaching units ready to be used in mathematics classrooms without any further modifications, i.e. in a way that teachers ask for. However, piloting of these materials has shown an interesting conclusion – considering the variety of classrooms, equipment, needs, abilities and skills of learners and teachers across Europe, it seems what is really needed are topics that can be adapted individually by different teachers for different learners and conditions. Instead of thousands of teaching units, we should look for rich sources of mathematics and alternative solving procedures that can then be adapted by individual teachers.

This paper shows one such multicultural topic (finger multiplication as a method of multiplication of a different cultural origin – though it is not really clear where exactly the method comes from – various sources speak of Russian, Gypsy or Chinese origin), which can easily be adapted for mathematics classrooms at very different levels, primary, upper secondary, teacher training.

A teaching unit with multicultural topic

The original topic was developed by the Norwegian partner of the project (Barbro Grevholm). The area of study is Multiplication from different approaches (history, culture, traditions, use of tools and books), the use of concrete tools in calculations, the use of early algebra for formulation of rules for multiplication and for proving mathematical results, different ways of proving in mathematics and mathematical reasoning. The aim of the unit was to make pupils reflect about the process of multiplication, realise the properties of multiplication and see links between multiplication and other areas of mathematics. Pupils may also reflect upon what they need to know by heart in mathematics and what can be reproduced with different tools or aids and may notice that mathematics is constructed and used by ordinary people in many parts of the world.

This teaching unit was piloted in two very different settings – 3rd grade of primary school in the Czech Republic using CLIL (Content and Language Integrated Learning, i.e. the lessons were conducted in English, thus making the language a "barrier" to everybody in the class), 18 year old upper secondary students in Austria and pre-service teachers of mathematics in the Czech Republic.

The teachers studied the Norwegian unit, analysed it and adapted it to suit the needs of their classrooms.

This paper only gives an overview of the piloting process. Its scope does not allow the full description of how the teachers conducted the lessons and to describe the course of the lessons and learners' activity and involvement; this will be presented in the oral presentation at the conference. It shows that the proposed activity can be successfully used in different settings, for different age groups, for both heterogeneous and homogeneous environment and different languages of instruction (e.g. CLIL – learning content through an additional language).

Piloting in Austria

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the 8th grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team.

The teacher conducted session 1 as described in the Norwegian material by handing out sheets containing a multiplication table from the year 1601 and started a group discussion about it. This discussion lasted about 12 minutes. The students were particularly interested in the aspects of why there was a need for such tables, whether such tables existed in their own cultures' history, and (mathematically) why these (shortened) tables were sufficient and contain the same basic data as the traditional, square-matrix shaped full multiplication tables they know. The information about the various aspects was partly given by the teacher, partly the students used internet resources to retrieve additional information.

Session 2 started with the introduction of the method of finger multiplication (5 minutes). Students were then asked to try the method out and find an explanation why it works (15 minutes). Students came up with several explanations and wanted to find out whether the method can be extended for numbers with more than one digit. They also were interested in whether this or other hand calculations were used historically. Two of the migrant students (from Turkey) reported about finger-based calculation methods from their own culture (12 minutes).

After the class, students were asked by the teacher about their experience with this teaching unit. Both the migrant and non-migrant students responded very positively. The migrant students particularly mentioned the chance of giving background information about their own culture that the other students did not know before. The non-migrant students commented positively on the various historical and cultural references that they not usually get during regular mathematics lessons.

Also, an interview was conducted with the teacher after the class. She particularly welcomed the possibility of having various anchor points for cultural references, and the opportunity to have the migrant students not only participate, but being a source of information for the other students.
The piloting clearly showed that students are interested in mathematics content from different cultures, and that the active participation of migrant students and the introduction of their cultural backgrounds can enrich the learning situation.

Piloting in Czech Republic

The teaching unit was piloted directly by one of the members of the project team and a co-author of this paper who, apart from being involved in research in the field of education, is a teacher. The teaching unit was piloted in the 3rd grade (9 year old pupils) in a primary school in Prague. It was piloted in a sequence of 4 lessons that were taught in the period of four months (about once in 6 weeks). Some of the lessons were video recorded and all the lessons were open to other teachers of the school as the school is now experimenting with the potential of CLIL in teaching in general and also in mathematics.

The teaching experiment was conceived as a sequence of lessons over a longer period of time. The multicultural background of the original Norwegian unit seemed to be the perfect environment for introducing the concept of learning mathematics through English to the learners who had had no former experience with it. Because the teaching unit was a CLIL unit and because it was planned for several lesson, the original Norwegian unit was supplemented by other activities – two different kinds of line multiplications whose origin is reported to be Chinese and games and other activities aiming at developing language skills (games with numbers, songs with numbers, What number am I?) or calculation skills (number centipedes). All the activities had two objectives – developing language and mathematical competence.

The lesson on finger multiplication was taught in September, 4 weeks after the beginning of the school year. The advantage at that stage was that the pupils had already mastered multiplication tables up to five but had not learned multiplication tables from 6 to 10. Thus it was an ideal situation for introduction of finger multiplication. Children who have memorized multiplication tables will find finger multiplication unnecessarily too difficult and time demanding. The teacher started by demonstrating the principle. The children were explained what number was represented by which finger and then shown by the teacher how the system works using fingers and whiteboard. The teacher then asked the children to do it but it turned out that at this point only very few understood. The teacher decided to demonstrate two other problems in front of the whiteboards but this time a pupil was always invited to assist and be showing it on their fingers. The whole class was saying the numbers out loud. After this the pupils were asked to work on their own. The teacher was monitoring, assisting individually to those pupils who needed help. One by one the pupils eventually grasped the principle. The teacher could see the "aha" effect when the pupils finally grasped the principle. In the subsequent lesson the teacher came back to the principle and could observe that the pupils found it relatively easy (but also inefficient as they know multiplication tables already). It was time to move to line multiplications of two and three digit numbers. The whole sequence of lessons, its outcomes and pupils' attitude are described in detail in (Moraová, Novotná, 2015).

It can be concluded that the teaching unit with cultural background proved to be very motivating and suitable for CLIL lesson. The uniqueness of the context contributed to the pupils' motivation and interest to be working on mathematics in English.

Figure 1. Finger multiplication

Conclusion

The fact is that mathematics classrooms are growing increasingly multicultural and multilingual. This growing diversity can be a chance to increase the quality of teaching (Slavin, 1994), but teachers will have to be trained to handle the situation. The differences in cultures and languages make the maths teaching-learning process harder for migrant and minority pupils than it is for majority pupils.

The presented study of adaptations of one topic (which could be understood as substantial learning environment in Wittmann's (1995) sense) clearly shows that it is possible to use the same environment in working with learners of different ages, knowledge of mathematics and sociocultural backgrounds. It is not needed to develop hundreds of teaching units in which teachers would painstakingly look for the one they can use without adaptations. On the contrary, pre-service and in-service teacher training should develop mathematics teachers' ability to work with materials creatively, to look for interesting topics and methods and modify them in such a way that makes the materials tailor-made for the needs of their own classroom.

The teaching experiment also proved that introduction of innovative solving methods whose roots come from different cultures is very motivating for the learners, brings fun into lessons and also develops flexibility of learners' thinking processes and awareness of the value of other cultures. The 3rd graders in the Czech Republic enjoyed the lesson, were active and involved. The teaching unit was piloted in 3 consecutive lessons and the teacher reported on the pupils' delight when they were told the teaching experiment would continue and not end after the first lesson. Doing something magic, learning a "trick", getting introduced to a way of work from other cultures was challenging, entertaining and effective. The Austrian students appreciated the chance to inquire into mathematics and its different cultural backgrounds, to look for the principles behind and for justification of the procedures. Although their mathematics levels were way beyond simple multiplications, they still found it interesting and satisfying to find out why these procedures work and what their limits are. Czech pre-service teachers grew aware of the potential of substantial learning environments as a rich source of teaching materials and experimented with how to adapt a teaching unit to meet different purposes.

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